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Dr. J. L. E. DREYER, PRESIDENT, in the Chair.

G. van Biesbroeck, Yerkes Observatory, Williams Bay, Wisconsin, U.S.A.;
Frank Hope-Jones, M.I.E.E., 124 Queen's Road, Richmond, Surrey; and
Kennet Pollock, 48A Stondon Park, Forest Hill, London, S.E. 23,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Francis John Bonham, 13 Dudley Road, Wimbledon, London, S.W. 19 (proposed by W. A. Parr);
John Anthony Carroll, B.A., Assistant Director, Solar Physics Observatory, Research Fellow of Sidney Sussex College, Cambridge (proposed by H. F. Newall);
The Hon. Lionel Guest, 26 Connaught Square, London, W. 2, and Sea Lane, Ferring, Goring-by-Sea, Sussex (proposed by R. T. A. Innes);
Claude Haines Hall, Jr., Yale University Observatory, New Haven, Connecticut, U.S.A. (proposed by F. Schlesinger);
Frederick James Hargreaves, F.C.I.P.A., Crosbie, Windermere Road, Coulsdon, Surrey (proposed by T. E. R. Phillips);
John Alun Lloyd, Telpyn, Ruthin, N. Wales (proposed by J. Silas Evans);

Herbert William George MacLeod, B.Sc., M.D., M.R.C.P., D.P.H., 20 Craven Hill, Lancaster Gate, London, W. 2 (proposed by W. A. Parr); and

Bruno Philip Henry de Roeper, Squadron Leader R.A.F., No. 1 Flying Training School, Netheravon, Wilts (proposed by W. J. S. Lockyer).

One hundred and ninety-three presents were announced as having been received since the last meeting, including, amongst others:—

Tycho Brahe, Opera Omnia, Tomus VII. (presented by Dr. Dreyer).

British (Terra Nova) Antarctic Expedition, 1910–1913:—C. Chree, Terrestrial Magnetism; H. G. Lyons, Miscellaneous Data; G. C. Simpson, Meteorology (presented by the Captain Scott Antarctic Fund).

Carte photographique du ciel: Zone $+14°$, Bordeaux Observatory (5 charts); Zones $-3°$ and $-5°$, San Fernando Observatory (31 charts): Zones $+5°$ and $+7°$, Toulouse Observatory (12 charts).

H. Dingle, Modern Astrophysics.

C. Lane Poor, Gravitation versus Relativity.

Washington, Hydrographic Office, Publication No. 203: The Sumner Line of Position (presented by Mr. H. B. Goodwin).

Mount Wilson Observatory, Seven Transparencies of Nebulæ and Portions of the Moon, taken with the 60-inch and 100-inch Reflectors.

Sproul Observatory, Swarthmore, Two Transparencies of the Total Solar Eclipse of 1923 September 10, taken at Yerbanis, Mexico.

G. Percy Wilkins, Twenty-four Charts of the Moon, with Index Map.

J. H. Worthington, Ten Transparencies of the Eclipse of 1923 September 10.

Cosmogonic Problems associated with a Secular Decrease of Mass.
By J. H. Jeans, D.Sc., Sec. R.S.

1. Some years ago I put forward the suggestion that the energy of stellar radiation is produced by a secular decrease in the star's mass, the mechanism possibly being that positive and negative electric charges fall together and annihilate one another, their energy being transformed into radiation. This conjecture has gained enormously in probability since Eddington has shown * that, as a matter both of theory and of observation, the radiation from a star is approximately a function only of the star's mass. It is now clear, both from theory and observation, that as a star's development proceeds, its mass must decrease. The star cannot, in any way known to us, store up more than a quite insignificant fraction of the energy liberated by this destruction of mass, so that it is

* *M.N.*, 84 (1924), 308.

natural to suppose that this energy escapes as radiation, the more so as this supposition solves the otherwise insoluble problem of the source of the energy of stellar radiation. Quite independently of the nature of the mechanism by which mass is transformed into radiation, the theory of relativity fixes the rate of exchange between these two forms of energy. The destruction of mass m sets free energy mC^2 where C is the velocity of light, this energy now being free to escape as radiation. The converse is also true; if energy E escapes as radiation, the mass of the radiator must decrease by E/C^2. For instance, from the known value of the solar constant, we can calculate that the sun must be losing mass at the rate of 4,200,000 tons a second. The mass of a giant star of absolute magnitude -5 must be decreasing at 10,000 times this rate.

Discussion of the problems of cosmogony has hitherto proceeded on the supposition that each star retains a constant mass throughout the whole course of its development. The conception of a secular decrease of mass brings new features into most of these problems; it is worth examining whether these carry with them the answers to any of the unsolved problems of cosmogony.

For instance, H. N. Russell has shown in a well-known paper* that, so long as outside disturbances are absent, the orbits of binary stars can only increase in size to a very limited extent. In obtaining this result, it was implicitly assumed that the masses of the components remain constant. If these masses are continually decreasing, the orbit can increase to a far greater extent than was calculated by Russell. This is best seen by passing to the limiting case in which both masses are suddenly reduced to zero; the orbits now reduce to straight lines which may be regarded as parabolas, or as ellipses of unlimited size. Among other problems, we shall have to examine whether this hitherto unconsidered possibility is capable of solving the mystery of the observed distribution of the orbits of binary stars.†

2. By an artifice as old as Newton, the problem of the gravitational motion of two bodies can be reduced to that of the motion of a particle about a fixed centre of force. It is readily seen that the same method can be used when the masses are in process of shrinkage; to determine the motion of either, we study the motion of a particle under a central force M/r^2, where $M = m^3/(m+m')^2$, m, m' being the masses of the two components whether constant or changing.

The motion of the particle will be governed by the usual equations

$$r^2\dot{\theta} = n \qquad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\ddot{r} - r\dot{\theta}^2 = -M/r^2 \qquad . \quad . \quad . \quad . \quad (2)$$

in which n is a constant but M varies with the time. From these equations we readily deduce

$$\frac{d}{dt}\left[\tfrac{1}{2}(\dot{r}^2 + r^2\dot{\theta}^2) - \frac{M}{r}\right] = -\frac{1}{r}\frac{dM}{dt} \qquad . \quad . \quad . \quad (3)$$

* *Astrophys. Journ.*, 31 (1910), 185.

† This possibility was suggested to me by Professor Russell.

showing that the energy of the particle in its orbit does not remain constant. Since dM/dt for an actual star will always be negative, the energy of the particle in its orbit continually increases. The rate of increase is represented by the right-hand member of equation (3) multiplied by the mass of the particle; this is equal to the rate at which work is done in removing the radiation emitted by the star out of the gravitational field of the particle.

3. From equations (1) and (2) we obtain in the usual way

$$\frac{d^2r}{dt^2} - \frac{n^2}{r^3} = -\frac{M}{r^2} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

an equation which does not admit of direct integration unless M is completely specified as a function of t.

Putting $Mr = \lambda$, the equation transforms into

$$\frac{1}{M^3}\frac{d^2}{dt^2}\left(\frac{\lambda}{M}\right) = \frac{n^2}{\lambda^3} - \frac{1}{\lambda^2} \quad . \quad . \quad . \quad . \quad (5)$$

If we now put

$$q = \int M^2 dt,$$

so that q is a function of t only, and

$$\frac{d}{dt} = M^2\frac{d}{dq},$$

we find that equation (5) becomes

$$\frac{d^2\lambda}{dq^2} - \frac{\lambda}{M}\frac{d^2M}{dq^2} = \frac{n^2}{\lambda^3} - \frac{1}{\lambda^2} \quad . \quad . \quad . \quad . \quad (6)$$

For this to be directly integrable we must have

$$\frac{d^2M}{dq^2} = \alpha^2 M \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where α^2 is a constant. Integrating equation (7) with respect to M and restoring t as variable, we find that equation (7) requires that

$$\frac{dM}{dt} = -M^2(\alpha^2M^2 + \beta)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (8)$$

where β is a constant of integration. The total rate of radiation L is, of course, connected with the rate of diminution of mass by the relation

$$L = -C^2\frac{dM}{dt} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

so that equation (6) becomes integrable if the emission L is proportional to $M^2(\alpha^2M^2 + \beta)^{\frac{1}{2}}$.

4. According to Eddington's theory, the emission L from stars of very great mass is proportional to $M^{1\cdot4}$, while that for stars of very small mass is proportional to $M^{4\cdot4}$. For stars of intermediate mass, the index of M is intermediate between the two extreme values of 1·4 and 4·4,

and Eddington's table * shows that there is a very considerable range of mass, corresponding roughly to the range between absolute magnitudes 5 and −1, for which the emission varies approximately as M^3. This can be represented by taking $\beta = 0$ in equation (8), so that

$$\frac{dM}{dt} = -\alpha M^3 \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Equation (4) now becomes

$$\frac{d^2\lambda}{dq^2} = a^2\lambda + \frac{n^2}{\lambda^3} - \frac{1}{\lambda^2} \quad . \quad . \quad . \quad . \quad (11)$$

whence, on integrating with respect to λ,

$$\frac{d\lambda}{dq} = \left(a^2\lambda^2 - \frac{n^2}{\lambda^2} + \frac{2}{\lambda} + A\right) \quad . \quad . \quad . \quad . \quad (12)$$

where A is a constant of integration.

The equation

$$a^2\lambda^2 + A + \frac{2}{\lambda} - \frac{n^2}{\lambda^2} = 0 \quad . \quad . \quad . \quad . \quad (13)$$

can have either one or three real positive roots in λ. The case of one real root corresponds to a quasi-hyperbolic orbit which is of no special cosmogonic interest. When there are three real positive roots, (if we reject the solution with λ larger than the largest root) the motion in λ determined by equation (12) will be an oscillatory motion between two adjacent roots, say $\lambda = \lambda_1$ and $\lambda = \lambda_2$. Since $\lambda = Mr$, it is clear that when the mass of the star is M the values of r will be oscillating between the values λ_1/M and λ_2/M. If M is not varying too rapidly, the orbit may be regarded as an ellipse, for which, if a, e denote its semi-major axis and eccentricity,

$$a(1-e) = \lambda_1/M$$
$$a(1+e) = \lambda_2/M.$$

From these equations

$$e = \frac{\lambda_2 - \lambda_1}{\lambda_2 + \lambda_1},$$

so that e depends only on the constants in equation (13) and so is independent of the time. We further have

$$a = \frac{\lambda_1 + \lambda_2}{2M} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

showing that, as M continually decreases, the dimensions of the orbit increase without limit, the product Ma remaining constant.

Thus the orbit of a particle is, to a first approximation, an ellipse which continually increases in size while keeping its eccentricity constant. In particular, a binary star which starts with a circular orbit must retain a circular orbit until it is disturbed by external influences. Clearly there is no explanation here of the observed high eccentricities

* *M.N.*, 84, 310.

of binary systems, while the theoretically calculated rate of increase of the dimensions of the orbit is too slow to offer an explanation of the observed wide separations of the components of later-type binaries.

5. If the radiation of the star is not strictly proportional to M^3, or to the more general form $M^2(a^2M^2+\beta)^{\frac{1}{2}}$, these conclusions need modification. In equation (11), a^2 may no longer be treated as a constant, so that equation (12) will not be a true integral. The value of λ will still oscillate between two values λ_1, λ_2, but these will show a secular variation with the time, so that there will be some variation in the eccentricity of the orbit.

The general problem is best attacked by returning to equation (3). If a is the semi-major axis of the elliptic orbit at any instant, the energy of the particle in its orbit is $-M/2a$, so that equation (3) becomes

$$\frac{d}{dt}\left(\frac{M}{2a}\right)=\frac{1}{r}\frac{dM}{dt}.$$

On averaging over a complete revolution of the particle, this becomes

$$\frac{d}{dt}\left(\frac{M}{2a}\right)=\frac{1}{a}\frac{dM}{dt} \quad . \quad . \quad . \quad . \quad (15)$$

it being easily shown that the average value of $1/r$ in an elliptic orbit is $1/a$. Equation (15) is now seen to have the integral

$$Ma = \text{constant} \quad . \quad . \quad . \quad . \quad (16)$$

The student of quantum-dynamics will notice that $(Ma)^{\frac{1}{2}}$, being proportional to the mean kinetic energy of the orbit divided by its frequency, is an "adiabatic invariant" and so necessarily remains constant throughout changes in M.

Thus, when the radiation is not restricted to vary as the cube of the mass, the major axis $2a$ still varies as $1/M$, while the eccentricity varies by an amount which depends on the extent to which the proportionality between M^3 and the emission fails.

6. Both from the general solution and the particular solution, it is clear that the conception of a secular decrease of mass brings no direct solution to the problem of the orbits of binary stars. But the time-scale to which this conception leads is such as almost of itself to provide a solution. Inserting the values appropriate to the sun $\left(M=2\times10^{33},\ -\frac{dM}{dt}=4{\cdot}2\times10^{12}\right)$, we find from the equation

$$\frac{dM}{dt}=-\alpha M^3 \quad . \quad . \quad . \quad . \quad . \quad (17)$$

that

$$\alpha = 5{\cdot}2\times10^{-88}.$$

Solving equation (17), it appears that the time required for a star to pass from mass M′ to mass M is given by

$$t=\frac{1}{2\alpha}\left(\frac{1}{M^2}-\frac{1}{M'^2}\right) \quad . \quad . \quad . \quad . \quad (18)$$

whence we find, if our sun ever had four times its present mass (corresponding roughly to a B-type star), the time since this period must be about $7{\cdot}1 \times 10^{12}$ years. The time since our sun had an enormously greater mass would be approximately the same, for the very massive part of the star's existence lasts only a short time. If we put $M' = \infty$ in equation (18), we obtain $t = 7{\cdot}6 \times 10^{12}$ years, but it must be remembered that our assumed law of radiation ($\propto M^3$) is not accurate for stars in the very massive state. In any case, since fission into binaries appears generally to occur somewhere about type B, or possibly earlier, we conclude that a binary which has been formed by fission and is now in the stage of development of our sun has probably been binary for something of the order of 7×10^{12} years. I have estimated * that, with the universe as it now is, a star may expect to encounter other stars at distances less than the radius of Neptune's orbit, at the rate of one encounter every 4×10^{12} years. Thus G-type dwarf binaries will on the average have experienced about two such encounters, and these would almost, although perhaps not quite, suffice to knock about the orbits of binary stars to the extent indicated by their observed eccentricities. For binaries which represent the relics of two separate condensations in the parent nebula, the argument is the same; the only difference is that we may perhaps estimate the life of the binary as somewhat longer.

It will be convenient to notice in passing some other consequences of the extended time-scale to which the conception of decreasing mass has led. With the universe as it now is, a star would expect to be deflected 7 degrees or more by encounters with another star about once in 4×10^{12} years,† so that a G-type dwarf would probably have experienced about two such encounters, as well as a number of less violent ones, in the course of its life. This goes some way at least towards explaining the tendency towards equipartition of energy which emerges from a study of the correlation between the masses and velocities of the star. Each time a light star encounters a massive one moving with a nearly equal velocity at a distance near enough to produce an appreciable deflection of orbit, the expectation is that the velocity of the massive star will be decreased, while that of the lighter star will increase. After an infinite number of such encounters there would be established the equipartition law, according to which the velocities of all the stars of any single type are dispersed about a mean which is inversely proportional to the square root of the mass. Halm has shown that the correlation between the masses and velocities of stars of different spectral types approximately conforms to this law.

Finally, it can be shown ‡ that a sun with a radius equal to that of Neptune's orbit would experience tidal pulls from passing stars of sufficient intensity to tear off planets, at the rate of about one in 3×10^{10} years. For a sun of radius equal to the earth's orbit the period is of the order of 10^{13} years. Thus we see that, even with the extended time-scale, the birth of satellites (on the tidal theory) is hardly a

* *Cosmogony and Stellar Dynamics*, p. 224.
† *Loc. cit.*, p. 226.
‡ *Loc. cit.*, p. 279.

normal event in a star's life, although it is now seen to be much less abnormal than has hitherto appeared to be the case.

7. Returning to the main problem, let γ denote the angle between the actual orbit of a particle at any point and a circular orbit through the same point, so that

$$\tan\gamma = \frac{\dot{r}}{r\dot{\theta}}.$$

Putting $r = \lambda/M$ as before, this becomes

$$\tan\gamma = \frac{1}{\dot{\theta}}\left[\frac{1}{\lambda}\frac{d\lambda}{dt} - \frac{1}{M}\frac{dM}{dt}\right] \quad . \quad . \quad . \quad (19)$$

In the particular case in which the emission is proportional to M^3, there are orbits (corresponding to circular orbits in the case of constancy of mass) for which λ retains a constant value, so that $d\lambda/dt = 0$. If we further put $n = r^2\dot{\theta} = \lambda^2\dot{\theta}/M^2$, equation (19) reduces to

$$\tan\gamma = -\frac{\lambda^2}{nM^3}\frac{dM}{dt} = \frac{\alpha\lambda^2}{n} \quad . \quad . \quad . \quad . \quad (20)$$

by equation (17). The value of $\tan\gamma$ is accordingly constant throughout the description of the orbit, and the orbit is the equiangular spiral

$$r = Ae^{\theta\tan\gamma} \quad . \quad . \quad . \quad . \quad . \quad (21)$$

In the more general case in which the emission is not proportional to M^3, equation (19) shows that the value of γ arises from two contributions, one depending on the fluctuations in λ, and the other on the secular changes in M. If we disregard the former, equation (19) shows that the value of γ will only be appreciable when $\frac{1}{M}\frac{dM}{dt}$ is appreciable in comparison with $\dot{\theta}$; that is to say, when the time-scale for the decrease in M is not enormously large in comparison with the period of an orbit. As we pass to larger and larger masses the time-scale, as measured by the reciprocal of $\frac{1}{M}\frac{dM}{dt}$, continually decreases, while the period of an orbit may either increase or decrease. To find instances in which γ is appreciable we must look for slow orbits about enormously massive centres.

8. It is to the spiral nebulæ that we naturally look for instances of such motions. The main body of the nucleus of a spiral is transparent, so that Eddington's theoretical calculation of emission in terms of mass will not apply, but the bright central (stellar) part of the nucleus must be thoroughly opaque, and if this is gaseous the theoretical formula ought to apply. If the mass of the opaque part of the nucleus is M times that of the sun, the emission ought to vary as $M^{\frac{3}{2}}(1-\beta)^{\frac{1}{2}}$, where $(1-\beta)$ is the ratio of radiation pressure to the total pressure. This is equal to 0·05 for the sun, so that $(1-\beta)^{-\frac{3}{2}} = 89{\cdot}4$; for the massive central part of the nebular nucleus $(1-\beta)$ cannot differ appreciably

from unity. Thus we may take the radiation from the nucleus to be $89{\cdot}4 \times M^{\frac{2}{3}}$ times that of the sun.

The sun radiates 1·9 ergs per second for each gramme of its mass, so that the nuclear star must radiate $170\,M^{\frac{2}{3}}$ ergs per gramme per second. The energy set free by the destruction of a gramme being 9×10^{20} ergs, it follows that if the nuclear star were to continue to radiate at its present rate, the time for which it would last would be

$$\frac{M}{dM/dt} = \frac{9 \times 10^{20}}{170\,M^{\frac{2}{3}}} \text{seconds} \quad . \quad . \quad . \quad . \quad (22)$$

Opik * estimates the mass of the central part of the Andromeda nebula to be $1{\cdot}8 \times 10^9$ times the mass of the sun. It is difficult to guess how much of this mass ought to be assigned to the central star; if we overestimate the mass of the central star by assigning to it the whole of this mass, the time-scale provided by equation (22) becomes

$$\frac{M}{dM/dt} = 3 \times 10^7 \text{ years} \quad . \quad . \quad . \quad . \quad (23)$$

Using Opik's estimate of 450,000 parsecs for the distance of the Andromeda nebula, Pease's spectroscopic velocity-determinations indicate a period of rotation at a distance of 150″ from the centre of

$$2{\cdot}3 \times 10^7 \text{ years.}$$

This period is of the same order of magnitude as that given by equation (23), so that tan γ, as given by equation 19, will be appreciable, and the orbits under gravitation (subject to the accuracy of the estimates we have used) must be spirals of quite appreciable angle.

Unfortunately, the results we have obtained do not accord numerically with the observed brightness of the central star. The radiation from this nucleus has been taken to be $89{\cdot}4 \times M^{\frac{2}{3}}$ times that of the sun; with the admittedly excessive value which we have used for M (namely, $M = 1{\cdot}8 \times 10^9$), this factor is found to be equal to about 10^{15}. Now, a star of luminosity 10^{15} times that of the sun, at our assumed distance of 450,000 parsecs would appear of magnitude -9, whereas the central part of the nucleus of the Andromeda nebula is only of magnitude about $+6$; even if we assume the light of the outer parts to arise entirely from reflection of light emitted by the central star, the magnitude of the latter, now represented by the magnitude of the integrated light of the whole nebula, is only about $+5$.

A central star of magnitude $+5$ at a distance of 450,000 parsecs would be of absolute magnitude -9, with a luminosity only 2×10^9 times that of the sun. This seems to suggest that for the largest masses of all the radiation may be proportional simply to M instead of to $M^{\frac{2}{3}}$, as we have imagined; this could be reconciled with Eddington's theory if we could suppose that the coefficient of absorption k approximated in very great masses to a constant limiting value k_0. With this smaller value for the emission of radiation, the time-scale for

* *Astrophys. Journ.*, 55 (1922), 406.

diminution of mass approximates to that for an ordinary star and the orbits differ only imperceptibly from circles. Although the conception of a secular decrease of mass fails to explain the spiral arms of nebulæ in terms of our present knowledge of the nebulæ, the attempt has been put on record as suggesting the possibility of equiangular and other spirals being described under ordinary gravitational forces.

9. The mass of the galactic system, of which our sun is a member, must be decreasing with a time-scale which is comparable with that of a typical star. If, as Poincaré suggested, the Milky Way may be regarded as an encircling belt of stars held in equilibrium by slow rotation, then the radius of this ring must be continually expanding, being connected with the mass of the galaxy approximately by the relation $Ma =$ constant. If, as is more natural, we regard the galaxy as a system of independently moving stars, the same relation $Ma =$ constant must apply to the radius of the orbit of a single star encircling the whole galaxy. Whichever way we regard the matter, it is found that the galaxy must be slowly expanding, approximately in accordance with the relation $Ma =$ constant. When our sun was a B-type star, and so of mass equal to four times its present mass, the mass of the whole galaxy must have been at least four times what it now is. At this time the radius of the galaxy would be less than a quarter of its present radius, and the stars packed together at least sixty-four times as closely as they now are. Thus the chances of close encounters between stars would be sixty-four times that estimated in § 6.

More precisely, let us assume the law of shrinkage of mass for each individual star to be that already assumed in equation (17), namely:

$$\frac{dM}{dt} = -aM^3.$$

If a star is of mass M_0 at time 0, its mass after time t will be

$$M = (2at + 1/M_0^2)^{-\frac{1}{2}},$$

whence it follows that the radius of the galaxy after time t will be given by

$$a = a_0(1 + 2atM_0^2)^{\frac{1}{2}}$$

and the density of stars ν by

$$\nu = \nu_0(1 + 2atM_0^2)^{-\frac{3}{2}}.$$

The total effect of the encounters of a star with its neighbours in any time from 0 to t will be proportional to

$$\int \nu dt = \frac{\nu_0}{aM_0^2}\left[1 - \frac{1}{(1 + 2atM_0^2)^{\frac{1}{2}}}\right] = \frac{\nu_0}{aM_0^2}\left(1 - \frac{M}{M_0}\right).$$

On further simplifying, this is found to be equal to

$$vt \times \frac{2M_0^2}{M(M + M_0)},$$

so that, so far as regards the effects of encounters between stars, the effect of early closer packing is to increase the time-scale by a factor

$$\frac{2M_0^2}{M(M+M_0)}.$$

If we take $M_0/M = 4$, this factor becomes equal to 6·4; with $M_0/M = 6$, it is equal to 10·3. If we multiply all the expectations mentioned in § 6 by the factor of 6·4 or 10·3, we find that the conception of a secular decrease of mass just about accounts for the observed orbits of binary stars and for the observed correlation between the masses of the stars and their velocities, while leaving the tidal genesis of planets a somewhat, although not excessively, rare event.

The Moon's Mean Longitude, Longitudes of Perigee and Node, Semi-Diameter and Parallactic Inequality derived from Occultations of Stars observed at the Royal Observatory, Cape of Good Hope, 1880–1922. By H. Spencer Jones, M.A., B.Sc., H.M. Astronomer.

The details of the occultations of stars by the Moon observed at the Royal Observatory, Cape of Good Hope, together with the resulting equations of condition, have been published for the period 1835 to 1880 in the *Cape Annals*, 1, part 4; for the period 1881 to 1895 in the *Cape Annals*, 2, part 3; and for the period 1896 to 1906 in the *Cape Annals*, 2, part 6. The details of the observations for the period 1907 to 1922 are in manuscript ready for publication. The whole series of observations have been reduced on a uniform plan, the places of the Moon being derived from Hansen's Tables, without Newcomb's corrections. The tabular places for the first series were supplied by Newcomb.

The positions of the Moon given in the *Nautical Almanac* from 1923 onwards are based upon Brown's Tables, which will henceforth be used in the reduction of the Cape Observations of Occultations. It therefore appeared desirable to discuss the observations up to the end of 1922 with a view to deriving the Moon's longitude, semi-diameter, eccentricity of orbit, and coefficient of the principal term in the parallactic inequality. The numbers of observations in many of the years before 1880 were very few; the discussion has therefore been restricted to the period 1880 to 1922.

The equations of condition have been published in the form

$$\frac{\sin\sigma}{\sin 1''} - S = (a)\Delta\alpha + (\delta)\Delta\delta + (\alpha')\Delta\alpha' + (\delta')\Delta\delta' + (T)\Delta T + (l)\Delta l + (\phi')\Delta\phi' + (p)p + (s)s,$$

where $\Delta\alpha$, $\Delta\delta$ are the corrections to α, δ, the Moon's tabular R.A. and Dec. in seconds of arc.

$\Delta\alpha'$, $\Delta\delta'$ are the corrections in seconds of arc to the R.A. and Dec. of the star occulted.

ΔT is the correction in seconds of time to the mean local time of observation.

Δl is the correction to the assumed longitude, l, in seconds of time.

$\Delta\phi'$ is the correction to the geocentric latitude, ϕ'.

$\sin\pi(1+p)$ is the sine of the true horizontal equatorial parallax of the Moon, the tabular value being $\sin\pi$.

$\sin\sigma(1+s)$ is the sine of the true semi-diameter of the Moon, the tabular value being $\sin\sigma$.

The longitude and latitude of the transit-circle adopted for the reductions are

$$-1^h\ 13^m\ 55^s{\cdot}00 \quad \text{and} \quad -33^\circ\ 56'\ 3''{\cdot}4.$$

The values of ϕ', $\log\rho$, and e are adopted as

$$\begin{aligned} \phi' &= -33^\circ\ 45'\ 26''{\cdot}26 \\ \log\rho &= 9{\cdot}9995512 \\ e &= 1/300. \end{aligned}$$

The parallax of the Moon is taken from Hansen's Tables, and the semi-diameter is deduced from it by the formula

$$[9{\cdot}43559]\times\pi.$$

The longitude of the transit-circle derived by telegraphic operations is $-1^h\ 13^m\ 54^s{\cdot}72$, requiring a correction of $+0^s{\cdot}28$ to the adopted longitude. Wireless time-signal comparisons indicate a slightly larger correction. The adopted correction is $+0^s{\cdot}30$.

For ϕ', ρ Hayford's spheroid of 1909 has been adopted, giving

$$\left.\begin{aligned} \phi' &= -33^\circ\ 45'\ 19''{\cdot}8 \\ \log\rho &= 9{\cdot}9995469 \end{aligned}\right\} e = 1/297.$$

As regards the correction to the horizontal equatorial parallax, observations at one place can only give the value of the local constant, $\pi + d\pi'$, where $d(\rho\pi) = \rho d\pi'$; $d\pi$, $d\pi'$ being the corrections to the true and local constants respectively.

Hence

$$d\pi' = d\pi + \pi d\log\rho.$$

The value of $d\pi'$ has been derived by the method used by Newcomb,* but with revised values of the constants. Newcomb pointed out that the parallax of the Moon referred to the mean radius of the Earth may be regarded as a known quantity not subject to correction. Assuming

* *Researches on the Motion of the Moon*, part ii. p. 40 (*Astronomical Papers of the American Ephemeris*, 9, part i., 1902).

the value 1/297 for the ellipticity, the correction to the local parallax may be computed and p eliminated from the equations of condition.

Adopting Helmert's value of gravity,

$$978{\cdot}046[1 + {\cdot}005302 \sin^2\phi - {\cdot}000007 \sin^2 2\phi],$$

we obtain, at mean latitude ($\sin^2\phi = 1/3$),

Apparent gravity	. .	979·767 cm./sec.^{2}
Centrifugal force	. .	2·253 ,,
Actual gravity	. .	982·020 ,,

Hayford's spheroid gives for the mean radius of the Earth 6371270 metres, whence, using the value of gravity just derived, the logarithm of the mass of the Earth expressed in cm.-sec. units is found to be 20·600572.

If a is the semi-major axis of the relative orbit of the Earth and Moon, n the mean motion in arc in one second,

$$a^3n^2 = \mathrm{E} + \mathrm{M}.$$

The mean sidereal motion of the Moon in a Julian century being 1336·85136 revolutions, $\log 1/n = 5{\cdot}574841$; adopting $\mathrm{E}/\mathrm{M} = 81{\cdot}530$, as used by Brown, it follows from the mass of the Earth previously found that

$$\log a = 10{\cdot}585183$$

where a is expressed in cms.

But

$$a \sin\pi = 1{\cdot}0009068\rho,$$

π being the constant of the Moon's horizontal parallax, and ρ the radius of the Earth corresponding to π.

If therefore, ρ is expressed in metres,

$$\sin\pi = [1{\cdot}415211 - 10]\rho,$$

and for the mean radius, 6371270 metres, corresponding to a horizontal parallax π_1, we find

$$\sin\pi_1'' = 3418''{\cdot}71.$$

The radius, ρ, of the Earth in any latitude ϕ', in terms of the mean radius ρ_1, and the ellipticity, e, is given by

$$\rho = \rho_1[1 + e(\tfrac{1}{3} - \sin^2\phi')],$$

so that the constant of the sine of the equatorial horizontal parallax is

$$\sin\pi_0'' = 3418''{\cdot}71(1 + \tfrac{1}{3}e).$$

There has been some difference of opinion as to the value of this constant actually employed by Hansen for the construction of his tables. Newcomb has used the value 3422″·09,* and also the value 3422″·07.†

* "Transformation of Hansen's Lunar Theory" (*Astronomical Papers of the American Ephemeris*, **1**, 78).

† *Compendium of Spherical Astronomy*, p. 136. Also *Researches on the Motion of the Moon*, part ii., p. 42.

Battermann * has discussed the question thoroughly, and concludes that the value $3422''\cdot12$ is the correct one. This value has been adopted in the present investigation.

With Hansen's parallax and an ellipticity e_0, the horizontal parallax for latitude ϕ' will be

$$3422''\cdot12(1-e_0\sin^2\phi').$$

The value resulting from the value of the equatorial horizontal parallax derived above is

$$3418''\cdot71[1+e(\tfrac{1}{3}-\sin^2\phi')].$$

The difference between these two expressions gives the correction to the adopted parallax. The correction to the local horizontal parallax is therefore

$$\Delta(\rho\sin\pi)=-3''\cdot41+1140''e+\sin^2\phi'[3''\cdot42e_0-3419''(e-e_0)].$$

Putting

$$e_0=\frac{1}{300}\qquad e=\frac{1}{297}$$

$$\Delta(\rho\sin\pi)=+0''\cdot32.$$

Also

$$\Delta(\log\rho)=-\cdot0000043$$
$$\sin\pi\Delta(\log\rho)=-''\cdot01.$$

The correction to be applied to the assumed value of the horizontal parallax is therefore $+0''\cdot31$.

The quantity p used in the reductions of the Cape occultations is consequently $+0\cdot31\sin1''/\sin\pi$ or $+0\cdot090\div10^3$. A correction of this amount has been applied to the equations of condition.

The star-places for the first and third series are taken, for the most part, from Newcomb's *Catalogue of* 1098 *Standard Clock and Zodiacal Stars* and *Catalogue of Zodiacal Stars*. Those for the second series are derived from Cape Observations, and to these mean corrections of $+0''\cdot32$ in R.A. and $+0''\cdot14$ in Declination have been adopted as a reduction to Newcomb's system.

With these corrections the equations of condition as published are reduced to the form

$$(\alpha)\Delta\alpha+(\delta)\Delta\delta+(s)s=\frac{\sin\sigma}{\sin1''}-\mathrm{S}=n \qquad . \quad . \quad \text{(i.)}$$

For further discussion it is necessary to transform the equations.

If λ, β are the ecliptic longitude and latitude of the Moon, we may write

$$(\alpha)\Delta\alpha+(\delta)\Delta\delta=(\lambda)\Delta\lambda+(\beta)\Delta\beta \quad . \quad . \quad . \quad \text{(ii.)}$$

where

$$(\lambda)=[1+(v\,.\,\alpha)](\alpha)+(v\,.\,\delta)(\delta)$$
$$(\beta)=(\beta\,.\,\alpha)(\alpha)+[1+(\beta\,.\,\delta)](\delta).$$

* *Beobachtungs-Ergebnisse der Königlichen Sternwarte zu Berlin*, No. 13, p. 11.

The quantities $(v.a)$, $(v.\delta)$, $(\beta.a)$, $(\beta.\delta)$ are extracted from the table given by Newcomb,* with arguments λ $(=v)$ and β. The quantities λ, β are derived from the *Nautical Almanac* for the G.M.T. of the observation of the occultation.

Transforming from ecliptic longitude and latitude to true longitude in the orbit, it is easy to show that

$$(\lambda)\Delta\lambda + (\beta)\Delta\beta = \mathrm{L}\Delta l_v + (\Omega)\sin i\Delta\Omega + (i)\Delta i \quad . \quad . \quad \text{(iii.)}$$

where l_v denotes the true longitude in the orbit, Ω the longitude of the ascending node, i the inclination of the orbit to the ecliptic, and where also

$$\mathrm{L} = (\lambda)[1 + (\mathrm{F}.l)] + (\beta)(\mathrm{F}.\beta)$$

$$(\Omega) = -\frac{1}{\sin i}[(\mathrm{F}.l)(\lambda) + (\mathrm{F}.\beta)(\beta)]$$

$$(i) = \sin(\lambda - \Omega)(\beta),$$

and where, in Newcomb's notation,

$$(\mathrm{F}.l) = \sin^2 i\{\sin^2(\lambda - \Omega) - \tfrac{1}{2}\}$$

$$(\mathrm{F}.\beta) = \sin i \cos(\lambda - \Omega).$$

$(\mathrm{F}.l)$, $(\mathrm{F}.\beta)$ are tabulated, with argument $u = \lambda - \Omega$, by Newcomb in table x., p. 50, *Investigation of Corrections to Hansen's Tables of the Moon.*

The correction to the true longitude in the orbit, Δl_v, may be expressed in terms of corrections, Δl_m, to the mean orbital longitude, Δe, to the eccentricity, $\Delta\Pi$ to the longitude of perigee and $\Delta\mathrm{P}$ to the coefficient of the principal term in the parallactic inequality, P, in the form

$$\Delta l_v = (1 + \mathrm{F})\Delta l_m + 2\sin g\Delta e - 2\cos g.e\Delta\Pi + (\mathrm{F}.\mathrm{P})\Delta\mathrm{P} \quad . \quad \text{(iv.)}$$

where g denotes the Moon's mean anomaly.

The quantity

$$\mathrm{F} = 2e\cos g + \tfrac{5}{2}e^2\cos 2g$$

is tabulated by Newcomb, with argument g, in table ix., p. 50, *Investigation of Corrections to Hansen's Tables of the Moon.*

Brown † gives for the principal parallactic terms

$$-125''{\cdot}154\sin\mathrm{D} + 18''{\cdot}023\sin(\mathrm{D} + g') - 18''{\cdot}609\sin(\mathrm{D} - g) - 8''{\cdot}466\sin(\mathrm{D} + g)$$

$$= -125''{\cdot}154[\sin\mathrm{D} - {\cdot}144\sin(\mathrm{D} + g') + {\cdot}149\sin(\mathrm{D} - g) + {\cdot}068\sin(\mathrm{D} + g)],$$

* *Investigation of Corrections to Hansen's Tables of the Moon*, table xi., p. 51. Also *Compendium of Spherical Astronomy*, table xxi., p. 432.

† *Tables of the Motion of the Moon*, sect. i., chap. i., List *ia*.

where $D = l_m - l_\odot$, $g = l_m - \Pi$, $g' = l_\odot - \Pi_\odot$,

$l_\odot$ denoting the longitude of the Sun, $\Pi_\odot$ the longitude of the Sun's perigee.

We may therefore adopt

$$(F.P) = \sin D - \cdot 144 \sin (D + g') + \cdot 149 \sin (D - g) + \cdot 068 \sin (D + g) \quad \text{(v.)}$$

As the coefficient of the principal term in the parallactic inequality used in the construction of Hansen's Tables, the value $-126''\cdot 45$, derived by Battermann,* is adopted, so that

$$P = 126''\cdot 45 + \Delta P.$$

Introducing the value of Δl_v from (iv.) into (iii.), it follows that

$$(\lambda)\Delta\lambda + (\beta)\Delta\beta = L'\Delta l_m + L \sin g \,.\, 2e\Delta\Pi - L \cos g \,.\, 2e\Delta\Pi + L(F.P)\Delta P + (\Omega)\sin i\Delta\Omega + (i)\Delta i + (\beta)\Delta\beta' \quad \text{(vi.)}$$

where

$$L' = L(1 + F).$$

The last term on the right-hand side is included in order to provide a means for the removal of the constant correction $-1''\cdot 00$ applied by Hansen to the latitude of the Moon on account of a supposed non-coincidence of the centres of gravity and figure of the Moon, which has no justification on theoretical or observational grounds.† To remove this term the constant correction $\Delta\beta' = +1''\cdot 00$ is applied.

The correction to the inclination, Δi, is small, and can be determined with but small weight from the occultations. The value derived by Newcomb ‡ is therefore adopted: $\Delta i = -0''\cdot 15$.

We then have

$$(i)\Delta i = -0''\cdot 15 \sin (\lambda - \Omega)(\beta) = \delta\beta_1(\beta)$$

and

$$(i)\Delta i + (\beta)\Delta\beta' = (1 + \delta\beta_1)(\beta).$$

The quantity $\delta\beta_1$ is tabulated by Newcomb in table x., p. 50, *Investigation of Corrections to Hansen's Tables of the Moon.*

The equations of condition (i.) now assume the form

$$L'\Delta l_m + L \sin g \,.\, 2\Delta e - L \cos g \,.\, 2e\Delta\Pi + L(F.P)\Delta P + (\Omega)\sin i\Delta\Omega + (s)s = n - (1 + \delta\beta_1)(\beta),$$

which is the form to which all the original equations of condition have been reduced.

The equation may for convenience be rewritten in the form

$$(x)\Delta x + (y)\Delta y + (z)\Delta z + (P)\Delta P + (\Omega)\sin i\Delta\Omega + (s)s = n'. \quad \text{(vii.)}$$

* *Beobachtungs-Ergebnisse der Königlichen Sternwarte zu Berlin*, No. 11, p. 19.
† Newcomb, *Researches on the Motion of the Moon*, part ii., p. 38.
‡ Newcomb, *Investigation of Corrections to Hansen's Tables of the Moon*, p. 36.

where

$$(x) = \mathrm{L}(1 + \mathrm{F}), \qquad (y) = \mathrm{L} \sin g, \qquad (z) = \mathrm{L} \cos g, \qquad (\mathrm{P}) = \mathrm{L}(\mathrm{F} \cdot \mathrm{P}),$$

$$\Delta x = \Delta l_m, \qquad \Delta y = 2\Delta e'', \qquad \Delta z = -2e\Delta\Pi,$$

and where s here denotes the original s multiplied by 10^3.

The various coefficients entering into equation (vii.) have been computed for each occultation, the values of the arguments being derived from the data in the *Nautical Almanac*.

In order to derive the quantities Δx, Δy, . . ., etc., with as great an accuracy as possible, corrections have been applied for terms in the Moon's motion omitted by Hansen or found on comparison with Brown's Tables to require correction. Battermann * has made a comparison between the terms of planetary origin used by Hansen in the compilation of his tables and the planetary terms according to Brown. He gives a list of the differences, Brown minus Hansen, which includes all terms with coefficients of at least $0''{\cdot}10$, and, in addition, terms with coefficients of or greater than $0''{\cdot}04$, whose periods approximate to the anomalistic period of revolution, and terms with coefficients of or greater than $0''{\cdot}02$, whose periods approximate to the synodical period of revolution. The inclusion of the small terms with these periods is necessary in order to derive accurate values of the eccentricity, of the longitude of perigee, and of the parallactic inequality.

As regards terms of solar origin, Brown † has given a comparison between his theory and Newcomb's *Transformation of Hansen's Lunar Theory*. The basis of the latter was not Hansen's *Tables de la Lune* but the *Darlegung der theoretischen Berechnung der in den Mondtafeln angewandten Störungen*. Brown's comparison therefore requires to be corrected by the addition of terms representing the difference between the *Darlegung* and the *Tables de la Lune*. The resulting terms which need to be applied to Hansen's tabular positions are given by Battermann.

Brown ‡ has given a detailed comparison between the terms depending on the figure of the Earth and the motion of the ecliptic according to his theory and as used in Hansen's Tables. The correction to be applied to Hansen depends upon the value assumed for the ellipticity of the Earth. The value 1/297 is adopted in the present discussion: for this value the correction to the inclination arising from these terms is negligible.

The list of the terms applied to Hansen is reproduced below in Table I. g, g' denote the mean anomalies of the Moon and Sun, ω and ω' the distances from the Moon's ascending node to the Moon's perigee and the Sun's perigee respectively. Π, V, E, M, J denote the mean longitudes of the Moon's perigee, Venus, the Earth, Mars, and Jupiter respectively, referred to the equinox of 1850·0. The longitude of the node, Ω, and the ecliptic longitude, λ, are referred to the equinox of the date. The terms have been numbered to agree with Battermann's list.

* *Beobachtungs-Ergebnisse der Königlichen-Sternwarte zu Berlin*, No. 13, p. 16.
† *M.N.R.A.S.*, 68, 148. ‡ *M.N.R.A.S.*, 74, 392.

TABLE I.

List of Terms: Brown – Hansen.

I. Planetary Terms in Longitude.

(*a*) Short Period.

1.	$+0''.13 \sin [g + (V - E) + 180^\circ]$
2.	$+0''.13 \sin [g - 3(V - E) + 180^\circ]$
3.	$+0''.16 \sin [g - 2(V - \Pi) + 179^\circ.6]$
4.	$+0''.13 \sin [g - (E - 2\Pi) - V]$
5.	$+0''.14 \sin [g + (E - J) + 1^\circ.0]$
6.	$+0''.19 \sin [g - 2(E - J) + 180^\circ]$
7.	$+0''.21 \sin [g + 2(\Pi - J) - 3(E - J) + 178^\circ.4]$
23.	$+0''.66 \sin [g - 2(E - \Pi) + 3(V - E) + 180^\circ]$
24.	$+0''.08 \sin [g - 2(E - \Pi) + 6(V - E) - 2E + 162^\circ.6]$
25.	$+0''.07 \sin [g + (2M - E) + 223^\circ.3]$
26.	$+0''.07 \sin [g - (2M - E) + 306^\circ.3]$
27.	$+0''.04 \sin [g + 2\Pi - 3(2M - E) + 162^\circ]$
28.	$+1''.14 \sin [g + 2(\Pi - J) + 180^\circ.3]$
29.	$+0''.44 \sin [g - J + 2(\Pi - J) + 7^\circ.5]$
30.	$+0''.15 \sin [D + (E - \Pi) - (V - E) + 180^\circ]$
31.	$+0''.14 \sin [D - (E - \Pi) + 2(V - E) + 180^\circ]$
32.	$+0''.16 \sin [D - (\Pi - J) + 359^\circ.0]$
33.	$+0''.02 \sin [D + (\Pi - J) - J + 116^\circ]$
34.	$+0''.05 \sin [D + (\Pi - J) + 180^\circ.9]$

(*b*) Middle-length Period.

8.	$+0''.28 \sin (V - E)$
9.	$+0''.12 \sin [2(V - E) + 180^\circ]$
10.	$+0''.35 \sin [E - 2(V - E) + 272^\circ.9]$
11.	$+0''.18 \sin [E - 3(V - E) + 271^\circ.7]$
12.	$+0''.10 \sin [(E - J) + 174^\circ]$
13.	$+0''.17 \sin [(E - J) - J + 298^\circ.5]$

(*c*) Long Period.

14.	$+0''.48 \sin (\omega - \omega' + 180^\circ)$ (solar term)
15.	$+0''.13 \sin [g - (E - 2\Pi) - 20(V - E) + 273^\circ.0]$
16.	$+0''.24 \sin [-(8V - 13E) + 313^\circ.9]$
17.	$+0''.85 \sin [-(18V - 16E - g) + 334^\circ.4]$

18. $+0^{\prime\prime}\cdot35 \sin[-(18V-16E-g)+(8V-13E)+33^\circ]$
19. $+0^{\prime\prime}\cdot11 \sin[-(18V-16E-g)-(8V-13E)+112^\circ\cdot0]$
20. $+0^{\prime\prime}\cdot24 \sin[2(\Pi-J)+0^\circ\cdot1]$
21. $+0^{\prime\prime}\cdot28 \sin[J-2(\Pi-J)+172^\circ\cdot5]$
22. $+0^{\prime\prime}\cdot13 \sin(J+286^\circ)$

II. Solar Terms in Longitude.

35. $+0^{\prime\prime}\cdot31 \sin(D-g)$
36. $+0^{\prime\prime}\cdot23 \sin[2(D-g)-2\omega]$
37. $+0^{\prime\prime}\cdot16 \sin[4(D-g)-2(\omega-\omega')]$
38. $+0^{\prime\prime}\cdot11 \sin[4(D-g)-2\omega-2(\omega-\omega')]$
39. $+0^{\prime\prime}\cdot28 \sin[g+2\omega+180^\circ]$
40. $-0^{\prime\prime}\cdot12 \sin(2D-g)$
41. $-0^{\prime\prime}\cdot19 \sin 2D$
42. $+0^{\prime\prime}\cdot25 \sin g'$

III. Terms due to the Motion of the Ecliptic and the Figure of the Earth.

43. $\Delta l_m = -0^{\prime\prime}\cdot97 \sin\Omega + 0^{\prime\prime}\cdot42 \cos\Omega$
44. $\Delta z = +0^{\prime\prime}\cdot42 \sin\Omega - 0^{\prime\prime}\cdot05 \cos\Omega$
45. $(\sin i\Delta\Omega) = +0^{\prime\prime}\cdot05 \sin\Omega - 0^{\prime\prime}\cdot04 \cos\Omega$

The short-period terms Nos. 1–34 and terms Nos. 35–42 were calculated for each occultation. Terms Nos. 8–13 were computed for 10-day intervals in the years in which occultations were numerous and interpolated for the time of occultation: in the years with comparatively few occultations these terms were computed for each occultation. The short-period terms represent corrections to the true longitude in the orbit and their sum after multiplication by L was applied as a correction to the residual in equation (vii.).

The long-period terms Nos. 14–22 were computed at intervals of 100 days and interpolation made for each occultation. These terms represent corrections to the longitude in the orbit, and therefore require to be multiplied by $L(1+F)$ and applied as a correction to the residual of the equation of condition. The term No. 43 was treated in the same manner.

The term No. 44 is a long-period disturbance of the quantity $z = -2e\Delta\Pi$. This correction was not applied in the formation of the equations of condition, but was introduced in the subsequent solution.

Two further corrections were applied. (i) In forming the original equations of condition the Hansen-Newcomb correction had been removed from the tabular places in order to secure a direct comparison with Hansen's Tables. This correction was reinserted; for the year 1880 the values were specially computed. (ii) A correction was intro-

duced to reduce the Hansen-Newcomb mean longitude (including the great empirical term) to the value adopted by Brown,* as corrected by Fotheringham,† and further corrected by Dyson and Crommelin.‡

Hansen's mean longitude is

$$335^\circ\ 43'\ 26''\cdot70 + 1336^r\ 307^\circ\ 53'\ 39''\cdot61 t_c + 13''\cdot30 t_c^2 + ''\cdot0135 t_c^3,$$

where t_c denotes centuries from 1800·0.

Newcomb's correction as used in the *Nautical Almanac* is

$$+1''\cdot14 - 29''\cdot17 t_c - 3''\cdot76 t_c^2 + 15''\cdot49 \sin(132^\circ t_c + 93^\circ\cdot9).$$

Hence the Hansen-Newcomb longitude (including the great empirical term) as used in the *Nautical Almanac* is

$$335^\circ\ 43'\ 25''\cdot56 + 1336^r\ 307^\circ\ 53'\ 10''\cdot44 t_c + 9''\cdot54 t_c^2 + ''\cdot0135 t_c^3 + 15''\cdot49 \sin(132^\circ t_c + 93^\circ\cdot9).$$

Brown's value (*Astron. Journ.*, No. 799), corrected for the secular acceleration found by Fotheringham, and with the constant term modified by Dyson and Crommelin, is

$$335^\circ\ 43'\ 25''\cdot10 + 1336^r\ 307^\circ\ 53'\ 13''\cdot82 t_c + 11''\cdot91 t_c^2 + ''\cdot0068 t_c^3 + 13''\cdot60 \sin(139^\circ t_c + 104^\circ\cdot2).$$

Hence the correction to be applied to the Hansen-Newcomb longitude is

$$-0''\cdot46 + 3''\cdot38 t_c + 2''\cdot37 t_c^2 - ''\cdot0067 t_c^3 + 13''\cdot60 \sin(139^\circ t_c + 104^\circ\cdot2) - 15''\cdot49 \sin(132^\circ t_c + 93^\circ\cdot9).$$

This term has been computed at yearly intervals and interpolated for each observation. After multiplication by L(1 + F) it has been applied as a correction to the residuals.

After the formation of the equations of condition, obviously discordant equations were examined. In a few instances errors were traced in the original reductions; in others an error of one or two minutes in the observed time was assumed. A small number of observations, which were marked by the observer as poor or uncertain or in which the observed time was apparently considerably in error, were rejected. Normal equations were formed from the remainder in small groups of one to four years, according to the number of observations, giving each occultation equal weight. These equations were solved for the unknowns, the probable error of one equation of condition and of each unknown being deduced in the usual manner from the squares of the residuals.

The values of the various unknowns derived in this way, together with the probable errors of one equation of condition and of each unknown, are given in Table II.

* Brown, *Tables of the Moon*, sect. 1, chap. i., p. 28; *Astron. Journ.*, No. 799.

† Fotheringham, *M.N.R.A.S.*, **81**, 125.

‡ Dyson and Crommelin, *M.N.R.A.S.*, 83, 361. The constant term of Brown's δT is changed from $-2''\cdot71$ to $-2''\cdot55$.

TABLE II.

Year.	$x=\Delta l$.		$y=2\Delta e''$.		$z=-2e\Delta\Pi$.		ΔP.		$\sin i\Delta\Omega$.		s.		P.E. of One Equation.
	″	″	″	″	″	″	″	″	″	″	″	″	″
1880	−2·18	±0·19											
81	−2·35	·35	−0·40	±0·24	−0·99	±0·21	+1·39	±0·26	+1·02	±0·27	−0·39	±0·19	±0·86
82	−2·04	·24											
83	−3·36	·44											
84	−1·60	·23											
85	−3·06	·28	−0·39	± ·19	−0·37	± ·21	+0·90	± ·27	+0·08	± ·30	−0·30	± ·15	0·86
86	−1·95	·20											
87	−1·87	·20											
88	−2·41	·19	−1·12	± ·17	−0·75	± ·25	+1·20	± ·33	−0·63	± ·26	−0·16	± ·18	0·95
89	−2·78	·24											
1890	−2·26	·23	−0·49	± ·20	−0·93	± ·22	+1·65	± ·29	+0·39	± ·24	−0·41	± ·18	0·85
91	−3·12	·20											
92	−3·57	·27											
93	−3·28	·20											
94	−3·90	·26	−0·32	± ·18	−0·72	± ·17	+0·62	± ·28	+0·83	± ·26	+0·06	± ·19	0·75
95	−3·71	·28											
96	−3·18	·14	−1·18	± ·13	−0·74	± ·17	+1·19	± ·20	+1·44	± ·19	−0·36	± ·12	0·56
97	−3·08	·11	−1·16	± ·10	−0·13	± ·12	+0·29	± ·19	+1·78	± ·18	+0·04	± ·14	0·57
98	−3·33	·10	−0·89	± ·16	−0·81	± ·13	+1·22	± ·25	+1·04	± ·19	−0·37	± ·20	0·89
99	−2·62	·11	−0·06	± ·16	−1·07	± ·13	+2·04	± ·26	+1·01	± ·17	−1·38	± ·21	0·76
1900	−2·31	·15	−0·33	± ·17	−0·95	± ·21	+1·89	± ·43	+1·17	± ·28	−0·87	± ·34	0·85
01	−1·81	·14	−0·65	± ·15	+0·15	± ·16	+1·40	± ·29	+0·93	± ·24	−0·44	± ·20	0·76
02	−1·39	·19											
03	−0·64	·22	−0·62	± ·18	−0·20	± ·18	+1·33	± ·44	+0·07	± ·26	−0·44	± ·27	0·67
04	+0·17	·27											
05	−0·18	·27	−1·36	± ·20	−0·57	± ·24	+1·17	± ·39	+0·21	± ·27	−0·29	± ·25	0·80
06	+0·23	·20											
07	+0·95	·19											
08	+1·28	·19											
09	+2·11	·21	−0·93	± ·15	−0·69	± ·16	+1·15	± ·28	+1·63	± ·18	−0·15	± ·17	0·72
1910	+2·20	·24											
11	+2·79	·29											
12	+3·04	·21											
13	+3·77	·27	−0·39	± ·17	−1·04	± ·18	+1·80	± ·31	+1·16	± ·28	−0·74	± ·21	0·78
14	+3·54	·22											
15	+3·34	·22											
16	+4·01	·23	−1·05	± ·25	−1·09	± ·23	+1·43	± ·28	+1·85	± ·26	−0·43	± ·21	±0·81

TABLE II—*continued.*

Year.	$x=\Delta l$.	$y=2\Delta e''$.	$z=-2e\Delta\Pi$.	ΔP.	$\sin i\Delta\Omega$.	s.	P.E. of One Equation.
	″ ″	″ ″	″ ″	″ ″	″ ″	″ ″	″
1917	+4·75± ·24						
18	+3·46 ·33	−0·22± ·21	−1·06± ·24	+1·63± ·26	+1·26± ·25	−0·38± ·18	±0·83
19	+4·68 ·24						
1920	+3·67 ·23						
21	+3·74 ·27	−1·24± ·19	−1·19± ·18	+1·04± ·41	+1·61± ·34	+0·08± ·34	±0·73
22	+3·78± ·21						

The quantities tabulated as z in the above table have been corrected by the application of the term No. 44.

The partial normal equations for the small groups used in the preliminary solution were then combined for the three periods 1880 to 1895, 1896 to 1906, 1907 to 1922, corresponding—except for the addition of the year 1880—to the three groups in which the occultations during this period are published in the *Cape Annals*. The values of the longitude corrections derived in Table I. were substituted in each group into the normal equations for y, z, ΔP, $\sin i\Delta\Omega$, and s, and a solution made for these quantities. The values so obtained were used to derive revised values for the longitude corrections from the normal equations for the x's, and with these revised values corrections to the preliminary values of y, z, ΔP, $\sin i\Delta\Omega$, and s were obtained. These corrections were all small, so that it was not necessary to proceed any further with the approximation.

The values derived from these three solutions for the corrections to the eccentricity, longitudes of perigee and node, parallactic inequality, and semi-diameter are given in Table III. The longitude corrections are not given, as definitive values of the longitude errors were derived from a final solution combining the three groups together. The probable errors given in this table have been assigned by assuming that the probable error for one equation of condition was $\pm 0''\cdot88$ for the period 1880 to 1895, and $\pm 0''\cdot80$ for the periods 1896 to 1906 and 1907 to 1922.

TABLE III.

	1880 to 1895.	1896 to 1906.	1907 to 1922.
	″ ″	″ ″	″ ″
$y=2\Delta e''$	−0·63 ±0·077	−0·70 ±0·051	−0·69 ±0·077
$z=-2e\Delta\Pi$	−0·77 ± ·078	−0·66 ± ·050	−0·86 ± ·077
ΔP	+1·289± ·111	+1·278± ·103	+1·451± ·153
$\sin i\Delta\Omega$	+0·24 ± ·115	+1·11 ± ·075	+1·50 ± ·109
s	−0·30 ± ·075	−0·48 ± ·075	−0·38 ± ·088
No. of occultation used	379	743	316

The agreement of the values resulting from the occultations observed during the three periods is very good, and in agreement with what would be expected from the assigned probable errors, except in the

case of the quantity $\sin i\Delta\Omega$, for which the values are somewhat discordant.

The values derived for ΔP when applied to the basic value $-126''\cdot450$ give the following values for the coefficient of the principal term in the parallactic inequality for the three periods:—

1880 to 1895 . .	$-125''\cdot161$
1896 to 1906 . .	$-125\cdot172$
1907 to 1922 . .	$-124\cdot999$

The coefficient of the principal term in the parallactic inequality, P, can be expressed in the form

$$-P = C \cdot \frac{a}{a'} \frac{E-M}{E+M}$$

where E, M denote respectively the masses of the Earth and Moon, a, a' are the semi-axes major of the orbits of the Earth and Moon, which for undisturbed elliptic motion correspond to the sidereal period of revolution. C is a constant which can be derived theoretically. If π is the solar parallax, p_0 the Moon's equatorial horizontal parallax, the fundamental constants in Brown's theory are:—

$$\frac{1}{a'} = \sin\pi \qquad \pi = 8''\cdot80549$$

$$\frac{1}{a} = \frac{1}{1\cdot00090768}\sin p_0 = 3419''\cdot596 \qquad \sin p_0 = 3422''\cdot540$$

$$p_0 = 3422''\cdot70$$

$$C = [4\cdot697301]$$

$$E \div M = 81\cdot53$$

$$\frac{\text{solar parallax}}{\text{lunar parallax}} \cdot \frac{E-M}{E+M} = \cdot00251287$$

$$-P = 125''\cdot154$$

With these figures the derived values for the solar parallax for the three periods are:—

1880 to 1895 . .	$8''\cdot8060 \pm 0''\cdot0079$
1896 to 1906 . .	$8\cdot8068 \pm \cdot0074$
1907 to 1922 . .	$8\cdot7944 \pm \cdot0109$

Weighting the three values in accordance with the probable errors, the resulting value is $8''\ 8040 \pm ''\cdot0048$, or, with sufficient accuracy,

$$8''\cdot804 \pm 0''\cdot005.$$

The value of the solar parallax derived in this way is dependent upon the value assumed for the ratio of the mass of the Earth to the mass of the Moon, but this ratio is now known with sufficient accuracy for its effect on the solar parallax to be practically negligible. To produce an alteration in the solar parallax of $0''\cdot001$, the ratio E/M

would require to be changed by 0·39. The value 81·53 which is adopted by Brown was derived by Hinks from the observations of Eros, and was assigned a probable error of ±0·05. The value derived from the lunar inequality is in very close agreement with Hinks' value. It is therefore very improbable that any subsequent revision of the value of the ratio E/M will change the value of the solar parallax deduced from the parallactic inequality by as much as 0″·0005.

The values derived for the other quantities by combining the above three solutions are as follows:—

$2\Delta e''$	$-0''\cdot68$	$\pm 0''\cdot037$	Solution I.
$-2e\Delta\Pi$	$-0\cdot73$	$\pm \cdot037$	
ΔP	$+1\cdot32$	$\pm \cdot068$	
$\sin i\Delta\Omega$	$+1\cdot01$	$\pm \cdot054$	
s	$-0\cdot39$	$\pm \cdot045$	

The probable error assigned to $\sin i\Delta\Omega$ has been derived in the usual manner from the probable errors assigned to the mean values in the three periods. It is much smaller than would be anticipated from the discordance between the three separate values. The question arises whether the differences in the three values are due (apart from the error in Hansen's mean motion of the node) entirely to accidental errors or to the omission of a periodic term. There appears to be some evidence of a term with a period of about nineteen years: thus, for instance, the values for 1883 to 1886 and for 1902 to 1906 are small; those for 1880, 1896 to 1901, 1915 to 1916 are large. The term $\sin i\Delta\Omega$ in the expression for Δl will give the error of the longitude of the node, provided that all terms in the Moon's motion with the period of revolution of the node have been included in forming the tabular places: the omission of any such term will result in a periodic variation in the deduced values of $\sin i\Delta\Omega$. Investigation has failed to reveal any term which would bring the results for the three periods into approximate uniformity. The derived values of $\sin i\Delta\Omega$ are to a certain extent dependent upon the value of the ellipticity assumed for the Earth's figure, the coefficient of $\sin\Omega$ in term No. 45 of Table I. being altered by change in the value of the ellipticity. But the extreme values for the latter may be taken as 1/293 and 1/298, and within this range the run in the values of $\sin i\Delta\Omega$ is not greatly altered. It must be concluded that the small value found during the first period is due to an accidental accumulation of small values in the early part of the series. The mean value derived from the whole solution may be accepted as giving the correction to the longitude of the node for an epoch approximately 1900·0.

The normal equations obtained for the three groups can be combined together to derive one general solution. As $\Delta\Pi$ and $\Delta\Omega$ are subject to secular variation arising from the corrections required by Hansen's mean motions, and the length of the whole period under discussion is forty-two years, it is necessary to introduce terms into the normal equations to allow for this, and also to derive two additional normal equations for

the secular variations. This can be done without a great deal of additional computation, using the method developed by Newcomb.*

The equations were again solved by successive approximation, the values of the longitude errors obtained in the previous work being used to derive preliminary values for the other unknowns. These values were then used to correct the longitude errors, and so on. The values of the unknowns thus derived have been substituted into the original equations of condition in order to derive the probable error of one equation of condition. In combining the partial normal equations for the separate years the inequality in z was removed in order to obtain the true value of z; the mean value of the inequality for each year was applied to the mean value of z, when substituting in the original equations of condition.

The solution depends upon 1438 occultations, in which two or more observations of the same occultation by different observers are treated as one occultation and given unit weight. The probable error of a single equation of condition, derived in the usual way, from the squares of the residuals, is $\pm 0''\cdot 81$.

The values so derived are :—

$y = 2\Delta e''$	$-0''\cdot 65$	$\pm 0''\cdot 037$	Solution II. Epoch 1900·0.
$z = -2e\Delta\Pi$	$-0''\cdot 71$	$\pm \cdot 036$	
ΔP	$+1''\cdot 291$	$\pm \cdot 061$	
$\sin i\Delta\Omega$	$+1''\cdot 01$	$\pm \cdot 053$	
s	$-0''\cdot 39$	$\pm \cdot 044$	
Δz	$-0''\cdot 64$	$\pm \cdot 36$	
$\Delta(\sin i\Delta\Omega)$	$+3''\cdot 45$	$\pm \cdot 54$	

The corrections Δz and $\Delta(\sin i\Delta\Omega)$ to the centennial mean motions of perigee and node have only a small weight, the period covered by this investigation being too short to enable reliable corrections to be derived. The large value obtained for the correction to Hansen's mean motion of the node is due largely to the accidental accumulation of several small values in the early part of the series, to which reference has already been made.

Instead, therefore, of deriving these two corrections from their normal equations, the corrections required to reduce Hansen's values of the mean motions to those adopted by Brown were substituted in the normal equations for y, z, ΔP, $\sin i\Delta\Omega$, and s. The resulting solution is :—

y	$= -0''\cdot 65$	$\pm 0''\cdot 037$	Solution III.
z	$= -0''\cdot 71$	$\pm \cdot 036$	
ΔP	$= +1''\cdot 296$	$\pm \cdot 061$	
$\sin i\Delta\Omega$	$= +1''\cdot 01$	$\pm \cdot 053$	
s	$= -0''\cdot 39$	± 044	

The probable errors are not appreciably altered.

* *The Elements of the Four Inner Planets and the Fundamental Constants of Astronomy*, 1895, pp. 49–52.

The Parallactic Inequality and the Solar Parallax.

The values resulting from these two solutions for the coefficient of the principal term of the parallactic inequality are:—

$$-125''.159 \pm 0''.061 \quad \text{Solution II.}$$
$$-125''.154 \pm \cdot061 \quad \text{,, III.}$$

The latter is identical with Brown's adopted value. The corresponding values of the solar parallax are:—

$$8''.806 \pm 0''.0044 \quad \text{Solution II.}$$
$$8''.805 \pm \cdot0044 \quad \text{,, III.}$$

The previous solution resulted in the value

$$8''.804 \pm 0''.005.$$

In Solutions II. and III. each equation of condition receives the same weight. In Solution I., on the other hand, each equation of condition received a slightly diminished weight in the first period, as the values derived from the small groups for the probable error of one equation, and given in the last column of Table I., were on the average larger for the first period. The three solutions are, however, in agreement within the assigned probable errors, and we may therefore adopt as the deduced value of the solar parallax the mean value

$$8''.805 \pm 0''.005.$$

The value derived by Hinks from the discussion of the Eros observations was

$$8''.806 \pm 0''.004.$$

The probable error of this determination of the solar parallax, as deduced from the internal accordance of the results, is satisfactorily small, and the value is in almost exact agreement with the Eros result. The question arises whether the determination can be regarded as free from systematic error. The parallactic inequality and the value of the adopted semi-diameter of the Moon are closely bound up with one another. In the case of meridian observations one limb can only be observed when the inequality is positive, and the other limb only when it is negative; it follows that, if the inequality is deduced from a comparison of its extreme observed effects on the Moon's position, the value so determined will be affected practically by the full amount of the error in the adopted semi-diameter. Owing to the effects of irradiation this quantity cannot be satisfactorily determined from meridian observations. Determinations of the solar parallax from the value of the parallactic inequality have therefore been generally regarded as subject to systematic errors of unknown amount. But, as pointed out by Newcomb,* the accurate determination of this inequality undoubtedly

* *Researches on the Motion of the Moon,* part ii., p. 13.

"affords one of the most precise methods of determining the solar parallax. In this determination the numerical divisor of this inequality is nearly 15, so that, possible theoretical uncertainty aside, the error of the parallax will be little more than one-fifteenth that of the inequality." The observation of occultations provides a means by which the systematic errors to which the meridian observations are liable may be avoided, subject to the proviso that observations of disappearance at the bright limb are restricted to stars not fainter than 3^m, and of reappearance at the dark limb to stars not fainter than 2^m. For fainter stars bright-limb disappearances would tend to be observed too early, and the reappearances too late. These conditions are satisfied by the occultations under discussion, which, with few exceptions, are dark-limb observations or observations during lunar eclipses. The possible theoretical uncertainties to which Newcomb referred, and which were emphasised by Gill * in his critique of this method of deriving the solar parallax, can be regarded, in view of the completion of Brown's theory of the motion of the Moon, as negligible. There is therefore no reason to believe that the value of the Moon's semi-diameter and the values of the parallactic inequality and of the solar parallax derived from the present discussion are affected by errors of a systematic nature.

The Eccentricity.

The values derived for y, the correction to the principal term in the Moon's ecliptic longitude, are respectively :—

$-0''\cdot68 \pm 0''\cdot037$	Solution I.
$-0''\cdot65 \pm \cdot037$	,, II.
$-0''\cdot65 \pm \cdot037$	,, III.

The value $y = 2\Delta e'' = -0''\cdot66$ is adopted as the correction.

Cowell † adopted the value $22640''\cdot168$ as the coefficient of the principal periodic term in the Moon's ecliptic longitude employed in Hansen's Tables. Newcomb ‡ derived the value $22640''\cdot15$. The value used by Cowell is adopted by Brown § as Hansen's value.

Applying the correction of $-0''\cdot66$ found above to this value, the resulting value of the principal term is

$$22639''\cdot51 \pm 0''\cdot04.$$

Newcomb ‖ obtained the value $22639''\cdot50$ from his discussion of occultations. Brown ¶ adopts $22639''\cdot55$ as his final value. The value $22639''\cdot50$ was used in the construction of the tables, but a means is provided for changing to the final value.

* "Determination of the Solar Parallax from Heliometer Observations of the Minor Planets Iris, Victoria, and Sappho," *Cape Annals*, **6**, part vi.

† *M.N.R.A.S.*, **64**, 163.

‡ *Transformation of Hansen's Lunar Theory*, p. 69.

§ *M.N.R.A.S.*, **74**, 421.

‖ *Researches on the Motion of the Moon*, part ii., p. 224.

¶ *Tables of the Moon*, sect. i., chap. i., p. 6.

The coefficient 22639″·55 corresponds to a value of the eccentricity of ·054900489, using the coefficient $2e - \frac{1}{4}e^3 + \frac{5}{96}e^5 + \frac{107}{4608}e^7$. The value resulting from the coefficient found in the present investigation is

$$e = \cdot 054900392 \pm \cdot 000000097.$$

This value is in agreement with Brown's within the limits of the assigned probable error.

Longitude of the Moon's Perigee.

The values derived for z from the three solutions are :—

$-0''\cdot73 \pm 0''\cdot037$	Solution I.
$-0''\cdot71 \pm \cdot036$	,, II.
$-0''\cdot71 \pm \cdot036$	,, III.

The value $z = -2e\Delta\Pi = -0''\cdot71 \pm ''\cdot036$ is adopted.

The resulting correction to Hansen's longitude of perigee is

$$\Delta\Pi = +6''\cdot46 \pm 0''\cdot33 \quad \text{(epoch 1900·0)}.$$

Hansen's * and Brown's † values for the longitude of perigee for epoch 1900 are respectively :—

Hansen : $\Pi = 334^\circ\ 19'\ 38''\cdot30 + 11^r\ 109^\circ\ 1'\ 50''\cdot08t_c - 36''\cdot25t_c^2 - ''\cdot037t_c^3$.

Brown : $\Pi = 334\ 19\ 46\cdot40 + 11\ 109\ 2\quad 2\cdot52t_c - 37\cdot17t_c^2 - \cdot045t_c^3$.

In these expressions t_c denotes the number of centuries, each of 36,525 days, from 1900·0. The origin of reckoning is the mean equinox of 1900·0.

Brown derives a correction to Hansen's mean motion of the perigee of $+12''\cdot44$ per century. In Solution II. a correction was derived to Hansen's mean motion of $+5''\cdot82 \pm 3''\cdot27$. The period covered by the occultations under discussion is not sufficiently long to enable this correction to be derived with much weight, and the difference between Brown's correction to Hansen's mean motion and that derived in the present investigation is only about twice the probable error of the latter.

The value of the correction to Hansen's mean longitude of perigee for the epoch 1900·0 is, however, derived with a comparatively small probable error. Since the difference Brown minus Hansen is $+8''\cdot10$ for 1900·0, the resulting correction to Brown's longitude of perigee for this epoch is

$$\Delta\Pi = -1''\cdot64 \pm 0''\cdot33,$$

a value which is five times its probable error. The longitude of the perigee derived by Newcomb in *Researches of the Moon*, part ii., p. 224, for epoch 1900·0 gives

$$\text{Brown} - \text{Newcomb} = +5''\cdot53.$$

* Newcomb, *Researches on the Motion of the Moon*, part ii., p. 224.

† *M.N.R.A.S.*, 75, 510; *Tables of the Moon*, sect. i., chap. i., p. 28.

Newcomb's value was corrected by Brown * so as to give the theoretical value of the secular acceleration at the epoch 1850 and to include a planetary term of long period neglected by Newcomb. After these corrections, the difference becomes for epoch 1900·0,

$$\text{Brown} - \text{Newcomb (corrected)} = +2''\cdot86.$$

The longitude of perigee derived above for the epoch 1900 thus falls approximately midway between Brown's value and Newcomb's value as corrected by Brown.

The Longitude of the Moon's Node.

The values derived for sin $i\Delta\Omega$ from the three solutions are:—

$+1''\cdot01 \pm 0''\cdot054$	Solution I.
$+1''\cdot01 \pm \cdot053$	,, II.
$+1''\cdot01 \pm \cdot053$	,, III.

and are identical. The resulting correction to Hansen's longitude of the node is

$$+11''\cdot28 \pm 0''\cdot59 \quad \text{(epoch 1900·0)}.$$

Hansen's † and Brown's ‡ expressions for the longitude of the node for epoch 1900 are:—

$$\text{Hansen}: \Omega = 259^\circ\ 10'\ 50''\cdot38 - 5^r\ 134^\circ\ 8'\ 43''\cdot21 t_c + 8''\cdot21 t_c^2 + ''\cdot007 t_c^3.$$

$$\text{Brown}: \Omega = 259\ 10\ 59\cdot79 - 5\ 134\ 8\ 31\cdot23 t_c + 7\cdot48 t_c^2 + \cdot008 t_c^3.$$

The value derived in Solution II. for the correction to Hansen's mean motion has very small weight. It is certainly considerably too large, due to several of the early years giving small values for the correction sin $i\Delta\Omega$.

The correction to Hansen's longitude for 1900·0 has a good weight. Since the difference for this epoch Brown minus Hansen is $+9''\cdot41$, the indicated correction to Brown is

$$\Delta\Omega = +1''\cdot87 \pm 0''\cdot59.$$

Newcomb's § final value, derived from his occultation discussion, but corrected by Brown || for a long-period planetary term, when compared with Brown's finally adopted value gives a difference

$$\text{Brown} - \text{Newcomb} = +2''\cdot41 \quad (1900\cdot0).$$

Newcomb's and the present investigations therefore indicate corrections to Brown's adopted value for 1900 of approximately equal amounts but in opposite directions.

* *M.N.R.A.S.*, **74**, 420.
† Newcomb, *Researches on the Motion of the Moon*, part ii., p. 224.
‡ *M.N.R.A.S.*, **75**, 510; *Tables of the Moon*, sect. i., chap. i., p. 28.
§ *Ibid.*, p. 224.
|| *M.N.R.A.S.*, **74**, 563.

The Semi-Diameter.

The value of the semi-diameter used in the Cape reductions of occultations is

$$[9{\cdot}43559] \times \pi,$$

where π is Hansen's tabular equatorial horizontal parallax. Battermann * has discussed the question as to the correct value to adopt for the value of the mean equatorial horizontal parallax as used by Hansen in the construction of his tables. He adopts

$$\sin \pi = 3422''{\cdot}12. \qquad \pi = 3422''{\cdot}28.$$

The correction to the semi-diameter resulting from the present discussion must be applied to that value of the semi-diameter which corresponds to this parallax, viz. 933″·06.

The three solutions give the following value for s :—

$-0''{\cdot}39 \pm 0''{\cdot}045$	Solution I.
$-0{\cdot}39 \pm {\cdot}044$	„ II.
$-0{\cdot}39 \pm {\cdot}044$	„ III.

The resulting value of the semi-diameter 933″·06 $(1 - 0{\cdot}39/10^8)$ is

$$932''{\cdot}70 \pm 0''{\cdot}04.$$

This may be compared with the following other determinations from occultation observations :—

L. Struve † . . .	$932''{\cdot}65 \pm 0''{\cdot}044$.
J. Peters ‡ . . .	$932{\cdot}49 \pm 0{\cdot}28$ (m.e.).
H. Battermann § . .	$932{\cdot}72 \pm 0{\cdot}15$.

The first of these determinations is based upon observations of occultations during two lunar eclipses, and the second upon observations of occultations of the Pleiades. In both instances the corrections to the assumed semi-diameter are derived from equations of condition of the form

$$(\alpha)\Delta\alpha + (\delta)\Delta\delta + (\pi)\Delta\pi + (r)\Delta r = \text{const.},$$

where Δr is the correction to the assumed value of the semi-diameter. The derivation of the semi-diameter from an equation of this form is not strictly legitimate. The values of the parallactic inequality and of the semi-diameter are closely bound up with one another, so that unless the correct value of the principal coefficient of the parallactic inequality is

* *Beobachtungs-Ergebnisse der Königlichen Sternwarte zu Berlin*, No. 13, p. 11.

† "Bearbeitung der während der totalen Mondfinsternisse 1884 October 4 und 1888 January 28 beobachteten Sternbedeckungen," Dorpat, 1893 ; also *Ast. Nachr.*, **135**, 175.

‡ "Berechnung der Coordinaten und des Halbmessers aus acht in den Jahren 1840 bis 1876 beobachteten Bedeckungen der Plejaden," *Ast. Nachr.*, **138**, 147.

§ "Beitrag zur Bestimmung der Mondbahn und des Mondhalbmessers aus Beobachtungen von Sternbedeckungen ; Vereinigung der Resultate der drei Berliner Reihen," *Beob.-Ergeb. der Kön.-Sternwarte zu Berlin*, No. 13.

employed in deriving the Moon's tabular place, any error in the coefficient will be to a large extent thrown into the deduced correction for the semi-diameter. During a lunar eclipse, an error of this nature will be minimised, as then sin D = 0, *i.e.* the principal term in the parallactic inequality is zero. The remaining terms in (FP) do not, however, vanish.

In the case of the observations during the two lunar eclipses discussed by Struve, the corrections to reduce from Hansen's to Brown's value of the principal coefficient of the parallactic inequality ($\Delta P = +1''\cdot30$) and to correct to Brown's value of the mean equatorial horizontal parallax ($\Delta\pi = -0''\cdot31$) have been roughly estimated, and the resulting correction to the assumed value of the semi-diameter is changed from $+0''\cdot12$ to about $-0''\cdot11$. It is apparent, therefore, that the correction to the semi-diameter obtained from any discussion of occultations in which the tabular places of the Moon are derived from Hansen's theory cannot be regarded as real unless a correction is applied for the incorrect value of the parallactic inequality employed by Hansen.

The third value, given above, is the mean result derived by Battermann from three series of occultations: in the reductions a term involving the correction to the parallactic inequality was introduced into the equations of condition; so that this value is free from the error just referred to.

The value derived in the present investigation is in satisfactory agreement with Battermann's mean value. Newcomb adopted for his occultation investigation a semi-diameter of $932''\cdot58$. This was based upon the results of Struve and Peters, referred to above, and upon the first two of Batterman's series. The results of the third series, which gave a value $932''\cdot81$, were not available to Newcomb. It would appear that Newcomb's adopted value requires to be increased by about $\frac{1}{10}$th second of arc.

The residuals of the equations of condition have been collected according to the magnitude of the stars and for the observations of disappearance and reappearance separately, in order to test whether there are any systematic differences.

The resulting mean residuals are given in the following table, in compiling which bright-limb observations and observations of occultations during lunar eclipses have been excluded.

TABLE IV.

Residuals arranged according to Star's Magnitude.

Mag.	Disappearances.		Reappearances.	
	No.	Mean.	No.	Mean.
m		″		″
>4·0	35	−0·19	29	−0·29
4·0–4·9	64	−0·17	53	+0·04
5·0–5·9	210	+0·10	149	+0·01
6·0–6·9	223	+0·07	114	0·00
7·0–7·9	87	+0·03	12	+0·18
8·0–8·9	165	−0·11	10	−0·06
9·0–9·9	163	−0·25	16	+0·77

The residuals are taken in the sense computed minus observed. The coefficient of the semi-diameter correction in the equations of condition is in all cases negative. It follows that if the residuals are interpreted as due to magnitude equation in the semi-diameter, the mean values for each group should represent, both for disappearance and reappearance, the excess of the group value over the mean value. The residuals for each magnitude group should then, accidental errors apart, agree in magnitude and sign for disappearances and reappearances.

The residuals may also be interpreted as due to magnitude equation in the star-places. Since for disappearances the coefficient of $\Delta\alpha'$ (the correction to the star's R.A.) in the equations of condition is positive for disappearances and negative for reappearances, the mean residuals should be approximately equal in magnitude but opposite in sign, for the two groups dealing with stars of the same magnitude.

To separate the two effects, the sum and difference of the mean residual for each magnitude group separately should be taken: the sum will give the magnitude effect in the semi-diameter and will eliminate magnitude equation in the star positions. The difference will give the latter and eliminate the former. Unfortunately the number of observed reappearances for stars fainter than $7^{m}\cdot0$ are so few that it is difficult to draw conclusions.

Table V. gives the weighted mean residual and the mean of the mean residuals for disappearances respectively, derived on the hypotheses that the residuals are due to magnitude equation in the semi-diameter and in the star-places respectively.

TABLE V.

Magnitude.	No.	Semi-diameter.		Star-places.	
		Weighted Mean.	Mean of R. and D.	Weighted Mean.	Mean of R. and D.
m		″	″	″	″
>4·0	64	−0·24	−0·24	+0·02	+0·05
4·0–4·9	117	−0·07	−0·06	−0·11	−0·10
5·0–5·9	359	+0·06	+0·05	+0·06	+0·04
6·0–6·9	337	+0·05	+0·03	+0·04	+0·03
7·0–7·9	99	+0·04	+0·10	+0·00	+0·05
8·0–8·9	175	−0·11	−0·08	−0·10	−0·04
9·0–9·9	179	−0·16	+0·26	−0·30	−0·51

The two columns headed "weighted mean" in this table should give the magnitude effects in the semi-diameter and star-places respectively, provided that the mean residuals arise from one cause only. If both are operating, the columns giving the mean of the R. and D. residuals separately should represent the two effects, but the probable errors of the values for the three last groups are large, on account of the small number of reappearances in these groups.

There is some evidence of a progressive change in both sets of residuals. In the columns headed "semi-diameter" a positive residual implies that the semi-diameter for that group is larger than the mean

value: it might be anticipated that for the faint stars the disappearances would be observed early and the reappearances late, and that therefore the mean residuals would tend to be positive for the faint stars. The residuals show a slight tendency in this direction, but with the exception of the first group, which is a comparatively small one, the residuals are not much larger than the probable error of the determination of the semi-diameter. It may be concluded that the magnitude effect in the semi-diameter, if present, is sufficiently small to be negligible in the mean.

In the columns headed "star-places" the residuals will be positive or negative according as the group value is smaller or larger than the mean value; in other words, according as the mean R.A. of the stars for the group is smaller or larger than the mean. There is a slight indication that the brightest and faintest stars are observed on the meridian somewhat late as compared with stars of intermediate magnitude. A similar effect has been detected for Newcomb's catalogue by Battermann. This magnitude equation, if real, is small in amount and negligible in the mean.

From this discussion it may be concluded that the observations are free from any serious magnitude effects.

The Mean Longitude.

The errors of mean longitude are given in Table VI. These values have been derived from the normal equations for the longitude errors, substituting $y = -0''\cdot65$, $z = -0''\cdot71$, $\Delta P = +1''\cdot30$, $\sin i\Delta\Omega = +1\cdot01$. The theoretical corrections to the mean motions of the perigee and node, $\Delta z = -1''\cdot36$, $\Delta(\sin i\Delta\Omega) = +1''\cdot07$, have been used. The inequality in z (term No. 44) has been taken into account. The probable errors assigned have been derived from the weights in the partial solutions in small groups, assigning a value of $\pm 0''\cdot81$ as the probable error of one equation of condition. These values sufficiently well indicate the probable accuracy of the errors of mean longitude for the various years. The observations are not distributed as uniformly as could be desired, several of the years near the beginning and end of the series being particularly weak.

The errors as given in the second column represent corrections to the mean longitude of Brown's Tables, after the latter has been corrected by the quantity $\delta T - 0''\cdot16$. The quantity called by Brown δT * is the correction to the value of the mean longitude employed in the construction of his tables, resulting from the correction to the secular acceleration found by Fotheringham, with consequential changes in the mean motion and long inequality.

The third column in Table VI. contains the sum of the difference of Brown and Hansen-Newcomb and of the quantity $\delta T - 0''\cdot16$ and represents the approximate correction necessary to reduce the errors given in column 2 to corrections to Hansen's tabular positions. The mean differences between Brown and Hansen-Newcomb have been tabulated by Brown,† the comparison including all the terms of fairly long period. The quantities in column 3 are derived from Brown's comparison. The resulting corrections to Hansen's tabular positions are

* *Astron. Journ.*, No. 799.
† *M.N.R.A.S.*, 73, 692; and *Astron. Journ.*, No. 799.

given in column 4. The next column contains the corrections to Hansen derived from the meridian observations at Greenwich,* each value being based upon observations on about 100 days.

The last column of the table gives the differences between the Cape and Greenwich corrections. In view of the fact that, apart from the period 1896 to 1900, the Cape values depend upon an average of only 24 occultations per year, the agreement is satisfactory. A portion of the differences may possibly be accounted for on the ground that the mean reductions from Brown to Hansen-Newcomb, as given by Brown, may appreciably differ, on account of the exclusion of the short-period terms, from the true value of the reduction which should be applied to reduce to the Hansen-Newcomb basis the error of mean longitude derived from a comparatively small number of occultations.

TABLE VI.

Correction to Mean Longitude.

Year.	Cape Occultations.	Reduction to Hansen's Tables with Newcomb's Corrections.	Correction to Hansen-Newcomb Tabular Positions.		Cape–Greenwich.
			Cape.	Greenwich. (T.C.)	
	″ ″	″	″	″	″
1880	−2·44±0·18	+3·14	+ 0·70	+ 1·75	−1·05
81	−2·61 ·33	3·20	+ 0·59	+ 1·95	−1·36
82	−2·24 ·23	2·50	+ 0·26	+ 0·99	−0·73
83	−3·40 ·41	1·92	− 1·48	+ 0·08	−1·56
84	−1·40 ·22	2·17	+ 0·77	+ 0·06	+0·71
85	−3·04 ·26	2·52	− 0·52	+ 0·21	−0·73
86	−2·03 ·19	2·55	+ 0·52	− 0·10	+0·62
87	−2·01 ·17	2·16	+ 0·15	− 0·13	+0·28
88	−2·48 ·16	2·29	− 0·19	− 0·47	+0·28
89	−2·79 ·23	2·53	− 0·26	+ 0·29	−0·55
1890	−2·49 ·22	2·19	− 0·30	+ 0·16	−0·46
91	−3·09 ·19	2·12	− 0·97	− 0·95	−0·02
92	−3·41 ·29	2·87	− 0·54	− 0·71	+0·17
93	−3·31 ·22	3·69	+ 0·38	0·00	+0·38
94	−3·88 ·28	4·22	+ 0·34	+ 0·92	−0·58
95	−3·45 ·30	4·46	+ 1·01	+ 1·33	−0·32
96	−3·50 ·20	5·18	+ 1·68	+ 1·67	+0·01
97	−3·39 ·16	5·74	+ 2·35	+ 2·46	−0·11
98	−3·33 ·09	5·14	+ 1·81	+ 2·58	−0·77
99	−2·90 ·12	4·47	+ 1·57	+ 2·07	−0·50
1900	−2·46 ·14	4·45	+ 1·99	+ 2·54	−0·55
01	−2·06 ·15	4·39	+ 2·33	+ 2·49	−0·16
02	−1·29± ·23	4·28	+ 2·99	+ 3·36	−0·37

* "Comparison of Hansen's Tables with Observations of the Moon made at the Royal Observatory, Greenwich, from 1851 January to 1922 December," *Greenwich Observations*, 1920, sect. G.

TABLE VI.—*continued.*

Year.	Cape Occultations.	Reduction to Hansen's Tables with Newcomb's Corrections.	Correction to Hansen-Newcomb Tabular Positions.		Cape–Greenwich.
			Cape.	Greenwich. (T.C.)	
	″ ″	″	″	″	″
1903	−0·57 ± 0·27	+4·36	+ 3·79	+ 3·11	+0·68
04	+0·07 ·27	5·05	+ 5·12	+ 4·07	+1·05
05	−0·01 ·27	5·41	+ 5·40	+ 5·42	−0·02
06	+·020 ·20	5·11	+ 5·31	+ 5·55	−0·24
07	+1·03 ·21	4·84	+ 5·87	+ 5·56	+0·31
08	+1·36 ·21	5·20	+ 6·56	+ 5·98	+0·58
09	+2·09 ·23	5·78	+ 7·87	+ 6·53	+1·34
1910	+2·23 ·27	6·11	+ 8·34	+ 7·97	+0·37
11	+2·93 ·30	6·53	+ 9·46	+ 8·40	+1·06
12	+2·95 ·22	7·61	+10·56	+ 9·89	+0·67
13	+3·73 ·28	8·63	+12·36	+11·71	+0·65
14	+3·67 ·23	8·84	+12·51	+12·55	−0·04
15	+3·48 ·22	8·80	+12·28	+12·66	−0·38
16	+3·96 ·23	9·17	+13·13	+13·64	−0·51
17	+4·53 ·23	9·28	+13·81	+14·02	−0·21
18	+3·27 ·32	8·98	+12·25	+13·21	−0·96
19	+4·49 ·23	8·76	+13·25	+12·52	+0·73
1920	+3·70 ·26	8·86	+12·56	+13·01	−0·45
21	+3·65 ·30	8·76	+12·41	+13·08	−0·67
22	+3·58 ± ·23	8·72	+12·30	+13·13	−0·83

The differences Cape minus Greenwich show several alternating sequences of positive and negative, but no periodicity is apparent.

Summary.

1. The occultations observed at the Cape Observatory during the period 1880 to 1922 have been discussed.

2. The corrections to the mean longitude of Brown's Tables as revised by Brown in *Astron. Journ.*, No. 799, are given in Table VI. The corrections required by the Hansen-Newcomb longitudes are also given, and compared with corrections indicated by the Greenwich meridian observations.

3. The derived correction for the epoch 1900·0 to Brown's longitude of the Moon's perigee is $-1''\cdot64 \pm 0''\cdot33$.

4. The derived correction for the epoch 1900·0 to Brown's longitude of the Moon's node is $+1''\cdot87 \pm 0''\cdot59$.

5. The eccentricity of the Moon's orbit is found to be ·054900392 ± ·000000097.

6. The coefficient of the principal term of the parallactic inequality is found to be 125″·15. The resulting value of the solar parallax is $8''\cdot805 \pm 0''\cdot005$.

7. The derived semi-diameter of the Moon is $932''\cdot70 \pm 0''\cdot04$.

The Mean Motions of the Moon's Perigee and Node and the Ellipticity of the Earth's Figure. By H. Spencer Jones, M.A., B.Sc., H.M. Astronomer.

1. From the discussion of the Cape observations of occultations during the years 1880–1922 * a correction was derived to Hansen's longitude of the perigee, for epoch 1900, of $+6''\cdot46 \pm 0''\cdot33$ and to Hansen's longitude of the ascending node, for the same epoch, of $+11''\cdot28 \pm 0''\cdot59$.

For the epoch 1900·0 the difference of longitudes of the perigee, Brown *minus* Hansen, is $+8''\cdot10$ and of the node is $+9''\cdot41$.

The derived correction to Brown's longitude of perigee is therefore $-1''\cdot64 \pm 0''\cdot33$, and to the longitude of the node is $+1''\cdot87 \pm 0''\cdot59$, for the epoch 1900·0. The corrections are respectively five and three times their probable errors, and can therefore be regarded as well established.

2. Brown's expressions for the longitude of perigee and node were derived from the discussion of the long series of Greenwich meridian observations of the Moon, and may be regarded as substantially correct for the weighted mean epoch of 1850. The corrections found for the epoch 1900 may therefore be attributed to the cumulative effect during fifty years of errors in the adopted mean motions of the perigee and node. Of the various terms which enter into the mean motions the one to which greatest uncertainty attaches is that arising from the figure of the Earth. A change in the value of the ellipticity from Faye's value 1/292·9 to Helmert's value 1/298·3 results in a change of 24″ per century in the mean motion of the perigee and of 21″ per century in the mean motion of the node.

We may therefore assume for the correction required by Brown's longitude of perigee the value

$$-1''\cdot64(1+2t_c),$$

and for the correction to the longitude of the node

$$+1''\cdot87(1+2t_c),$$

where t_c denotes centuries from 1900. These expressions vanish for 1850 and have values $-1''\cdot64$, $+1''\cdot87$ respectively for 1900.

The correction to the mean motion of the perigee is therefore $-3''\cdot28$ and to the mean motion of the node $+3''\cdot74$.

3. The terms depending upon the figure of the Earth are proportional to $(\epsilon - \frac{1}{2}\phi)$, where ϵ denotes the ellipticity of the Earth's figure and ϕ is the ratio of the centrifugal force at the equator to gravity at the equator. ϕ has the value ·003468. Writing $1/\epsilon = k$, if C is the coefficient of any term depending upon the figure of the Earth

$$C \propto \left(\frac{1}{k} - \tfrac{1}{2}\phi\right),$$

* *M.N.R.A.S.*, 85, 11.

and the change δC in C due to change δk in k is

$$\delta C = -C\delta k/k(1-\tfrac{1}{2}k\phi),$$

or, since $\frac{1}{2}k\phi = 0{\cdot}5$, approximately

$$\delta C = -2C\delta k/k.$$

The value of k adopted by Brown is 294·0.

Corresponding to this value the centennial motion of the perigee arising from the figure of the Earth terms is $+651''$. Putting $\delta C = -3''{\cdot}3$, we derive $\delta k = +0{\cdot}8$.

The centennial motion of the node arising from the figure of the Earth and corresponding to $k = 294{\cdot}0$ is $-609''$. Putting $\delta C = +3''{\cdot}7$, we derive $\delta k = +0{\cdot}9$.

Hence we obtain the following values of the ellipticity :—

From the motion of the perigee	1/294·8
„ „ node	1/294·9.

If the correction to the mean motions could without any uncertainty be attributed to the figure of the Earth terms, the probable error of the value of δk derived from the perigee would be $\pm 0{\cdot}2$, and of that derived from the node would be $\pm 0{\cdot}3$. But the portions of the mean motions which depend upon the direct and indirect actions of the planets are also subject to slight uncertainties, and it is therefore impossible to assign a probable error to this determination of the ellipticity of the Earth's figure.

In view, however, of the small probable errors attaching to the determination of the corrections to Brown's longitudes of the perigee and node and of the smallness of the corrections themselves, it may be concluded that Brown's adopted value of the ellipticity, 1/294, is not very far from the true value. The value 1/294·8 would provide better agreement with the occultation observations.

The value 1/297 was assumed for the figure of the Earth terms in the discussion of the occultations. The period covered by the discussion is approximately that of two revolutions of the Moon's node, and the change in the value of the ellipticity would not produce any appreciable change in the corrections to the longitudes of the perigee and node derived for the mean epoch 1900·0.

4. Applying the corrections $-1''{\cdot}64(1+2t_c)$ and $+1''{\cdot}87(1+2t_c)$ to Brown's expressions for the mean longitudes of the perigee and node respectively, the resulting values of the longitudes, epoch 1900, become

$$\Pi = 334^\circ\ 19'\ 44''{\cdot}76 + 11^r\ 109^\circ\ 1'\ 59''{\cdot}24t_c - 37''{\cdot}17t_c^2 - ''{\cdot}045t_c^3,$$
$$\Omega = 259^\circ\ 11'\ 1''{\cdot}66 - 5^r\ 134^\circ\ 8'\ 27''{\cdot}49t_c + 7''{\cdot}48t_c^2 + ''{\cdot}008t_c^3,$$

where t_c denotes centuries from 1900.

Royal Observatory, Cape of Good Hope:
1924 *November* 4.

On the Spectrum of Germanium. By Joseph Lunt, D.Sc.
(Plates 1, 2.)

(*Communicated by H.M. Astronomer at the Cape.*)

Owing to the scarcity of the metal germanium, its spectrum has hitherto not been fully investigated; and very little was known about the visual region and the variations in the spectrum at different temperature levels.

The recent discovery of a rich and plentiful source of the metal in the germanium-gallium mineral "Germanite" from South-West Africa has made it possible to investigate the spectrum more fully.

A spectrographic analysis of the mineral has been published by the writer.* Mr. Kriesel, chemist at the Tsumeb mines, very kindly prepared and presented specimens of the metal for this investigation.

As would be expected, the metal was not spectroscopically pure, but contained traces of Fe, Mn, Pb, Ag, Zn, Cu, Ca, Mo, As, Sn, and Na as impurities, the lines of iron being the most numerous.

I. *H_γ Region*, λλ 3938–4743.—The first photographs were of the spark between metallic terminals in air at atmospheric pressure, using an 18-in. Apps coil, a capacity † of 0·06 μF, a self-induction of 248 μH, and a small spark-gap in the secondary circuit (cool spark).

The image of the spark was placed *across* the slit, and the only lines which appeared were the arc lines ‡ 4227 and 4686, whilst the neighbouring lines recorded by Exner and Haschek were very unexpectedly absent. The missing lines appeared, however, on placing the image of the spark *along* the slit, but they were confined to the neighbourhood of the poles, and did not, like the other two lines, occur throughout the spark (Pl. 1, strips II. and III.).

Exner and Haschek's lines 4179, 4261, and 4292 were thus shown to be enhanced lines, which only appeared in the transverse spark when the terminals were closer together and no self-induction used in the secondary circuit (Pl. 1, strip I., vertical hot spark). The same photographs showed that Exner and Haschek's pair of lines at 4088, 4099 were in reality wide, very diffuse doubles at 4085·7, 4088·2 and 4098·1, 4100·2, and that they also were enhanced lines suppressed by self-induction (Pl. 2, strip VI.).

The only other lines to note in this (H_γ) region are Exner and Haschek's line 4602·35, which appears not to belong to germanium—it is possibly a strong lithium line,—a very diffuse line at 4744, where their measures terminate (Pl. 1, strip VIII.), and a diffuse enhanced line at 3938.

With the exception of the double pair of diffuse lines referred to,

* *South African Journal of Science*, 20, 157; see also *Journal of the Chemical Society* (London), 125, 818.

† The values of the capacities and self-induction are given in microfarads and microhenrys, μF and μH.

‡ Papish gives a photograph of these lines in *Chemical News*, 124, 2; Eder and Valenta show only a feeble spectrum of germanium in their Atlas.

and the line 4602 (Li?), Kobb* had recorded all these lines (except 3938) in 1886.

Rowland and Tatnall† had recorded only 4227, Gramont only 4227 and 4686 with a ? to a line at 4743.

Beyond these lines to the red, Kobb and Gramont recorded only nine and seven lines respectively, and their early wave-lengths are not very accurate. In view of the numerous impurity lines in the spectrum of the metal, due to the elements referred to, and their blending in some cases with the lines to be measured, it was decided to seek confirmation of the origin of the lines and better wave-lengths by working with germanium tetrachloride in vacuum-tubes.

Professor Smeath Thomas, of the University of Cape Town, very kindly prepared and presented an ample supply of the carefully distilled liquid chloride for this purpose.

The gaseous chloride, like tin tetrachloride, dissociates and gives the spectra of its two constituents without the use of capacity in the secondary circuit. The spectra have been valuable, inasmuch as at the low pressure —10-12 mm. of $GeCl_4$—lines which were very diffuse and difficult to measure in the spectra of the metal spark became quite sharp and could be placed with accuracy. They revealed also some extraordinary shifts of lines, which will be referred to later, and incidentally extended our knowledge of the spectrum of chlorine in the visual region, hitherto only imperfectly investigated.

All the lines previously referred to as due to germanium have been confirmed in the vacuum-tube spectra of the chloride, except 4292, about which there is no doubt, which is blended with a chlorine line. Eder and Valenta's strong chlorine line at 4226·58 does not appear even when Ge 4226·76 is faint and sharp.

II. *D Region*, λλ 4743–7147.—This region has yielded much that is new, and perhaps of greatest interest is a group of lines of peculiar behaviour, on the blue side of the D lines, which appears to exist only through a short temperature range. The lines are strong when self-induction is used with the metal spark; but at the higher temperature obtained when self-induction is eliminated they vanish completely, when 5894 is made as strong as before (Pl. 2, strips I. and II.). These lines are placed together in Class II. In Class I. are included the arc lines and the spark lines which are not suppressed by self-induction. The enhanced lines are placed in Class III.

In the spectrum of germanium chloride, with an uncondensed discharge passed through a short end-on capillary tube, the lines of Class II. are strong and sharp; but when capacity ‡ and a spark-gap are introduced the luminosity of the capillary becomes very greatly enhanced, and when 5894 is stronger than before these lines vanish, just as they do in the hotter spark (Pl. 2, strip III.).

The spark lines of Class II. are more diffuse and sensibly displaced to the red with respect to the sharp vacuum-tube lines, and some are shaded on the red side. Nine lines show a mean displacement of

* Kayser's *Handbook*, **5**, 471.

† *Astrophysical Journal*, **1**, 149.

‡ A 6-in. coil and a capacity of 0·0025 μF in the secondary was used here.

0·40 Å. (see Table II.), whereas impurity lines measured in the same stretch of spectrum, due to copper, molybdenum, sodium, and nitrogen, do not share this displacement, but agree, within a few hundredths, with their tabular positions.

In addition to the lines of Class II., there are three pairs of lines with a constant frequency-difference of 360; the first pair was recorded by Kobb, and consists of the strong lines in the yellow and orange, the former occurring between the D lines (Pl. 1, strip XV.; Pl. 2, strips IV. and V.).

The first (more refrangible) line only of the next pair—which is on the violet side of H_α—was recorded by Kobb. The third pair is in the extreme red (Pl. 1, strips XVI. to XVIII.).

In addition to these are four pairs of enhanced lines, two with $\Delta\nu$ 459 and two others with $\Delta\nu$ 169. The wave-lengths and frequency differences are given below.

Table I.

λ Rowland.	I.A. corrected to Vacuum.	ν.	$\Delta\nu$.	Class.
5893·63	5895·01	16963·5	359·6	I.
6021·25	6022·67	16603·9		I.
6336·4	6337·91	15778·1	360·2	I.
6484·4	6485·95	15417·9		I.
6966·8	6968·45	14350·4	359·5	I.
7145·8	7147·49	13990·9		I.
4179·22	4180·21	23922·2	458·8	III.
4260·94	4261·95	23463·4		III.
5106·98	5108·20	19576·4	458·8	III.
5229·54	5230·79	19117·6		III.
4260·94	4261·95	23463·4	168·7	III.
4291·80	4292·82	23294·7		III.
5210·40	5211·65	19187·8	169·3	III.
5256·76	5258·03	19018·5		III.

Paulson * recorded three triplets with $\Delta\nu$ 1416, 903 and two pairs with $\Delta\nu$ equal to the sum of the above, all—with the exception of the arc pair 4227, 4686—in the ultra-violet. No typical series has been discovered.†

Photographs of the spectrum of germanium have been published by Exner and Haschek ‡ and by Eder and Valenta in their Atlas. Two plates accompany this paper.

The measured wave-lengths of the germanium lines are given in Table II.

The lines given are almost exclusively confined to those found in

* *Annalen der Physik*, 45, 426 (1914).

† Fowler, *Report on Series in Line Spectra*, p. 163.

‡ *Sitzungsberichten der Kaiserl. Akademie der Wissenschaften in Wien*, 108, pls. i. and ii. (1899).

the spark spectrum of the metal in air at atmospheric pressure, which have been confirmed in the spectra of the chloride in vacuum tubes.

The spectrum of the chloride shows numerous lines—which may belong either to germanium or chlorine (or mercury)—which have not yet been traced to their origin, as our knowledge of the spectrum of chlorine in the visual region is still very imperfect. Many other lines have already proved to be unrecorded chlorine lines, as they occur in the vacuum-tube spectrum of tin tetrachloride.

The work on the spectrum of chlorine is being continued. The origin of other lines may be found in the vacuum-spark spectrum of germanium—not yet investigated—which, as Anderson * shows in the case of calcium, may be expected to yield "many lines not given by the arc, or by the spark in air."

Variability of the Wave-Lengths.

A large discrepancy between the wave-lengths of the lines of the more refrangible pair in the red was found on comparing the vacuum-tube lines with the spark lines, amounting to no less than 1·5 Å. It was found that the variation could be obtained in consecutive exposures with the vacuum tube, using in the first case the direct discharge of the 6-in. coil without capacity, and in the second a capacity of 0·0025 μF, no self-induction, and a spark-gap.

These displacements are regarded as temperature shifts; numerous cases occur in the spectrum of chlorine which form the subject of a separate communication. Experiments already made indicate that the large discrepancies between the wave-lengths of chlorine lines given by Eder and Valenta and by Exner and Haschek are due to the differences of temperature used by them, the former using the gas in vacuum tubes, the latter the spark between carbon electrodes impregnated with potassium chloride. These results have an important bearing on current astrophysical investigations.

The pair of diffuse spark lines 4742, 4815, which are sharp in the vacuum-tube spectrum of the chloride with uncondensed discharge, show a similar temperature shift (Pl. 1, strip VII.) to that given by the red pair and not shared by Ge 4686 or the neighbouring chlorine lines.

Rowland,† in his report of progress in spectrum work in 1891, included germanium in the list of elements present in the sun; the number of lines (1 or 2) is not stated, nor are their wave-lengths.

In 1894 he published wave-lengths ‡ of 27 lines, mainly in the ultra-violet, obtained from the rare germanium mineral argyrodite. Only six of these are in the region covered by his preliminary table of solar spectrum wave-lengths. In these tables (1896) none of the solar lines are ascribed to germanium, and it would appear that Rowland abandoned the idea of the presence of that metal in the sun.

* *Astrophysical Journal*, **59**, 76.
† *Physical Papers*, pp. 522, 523.
‡ *Astrophysical Journal*, **1**, 149 (Rowland and Tatnall).

The following figures elucidate this point:—

	λ.	Intensity.	Intensity in Sun.	Rowland's nearest Solar Lines.			E. and H.'s Intensity Spark.
Rowland and Tatnall *	3039·198	100 r	5	3039·170	2	No origin	20
	3269·628	60	2	3269·627	0	No origin	10
	4226·724	7 s	...	4226·724	0	Sr?	50
E. and H. Lunt.	4686·09	...	...	4686·028	00 N	No origin	10
	4686·08						10

The last two lines (arc lines) might be expected if the metal were present. There is, therefore, no evidence of the presence of germanium lines in the solar spectrum.

Plates 1 and 2 show enlargements of the spectra dealt with in this paper. The spectra were obtained with a four-prism laboratory spectrograph made by Hilger to my design. Two shutters placed alternately in front of the slit are used to obtain two spectra one above the other and in contact; the measures are made near the point of contact. The slightly enlarged image of the source of light is focused on the slit by a lens fixed in the optical axis of the collimator, and a right-angled prism can be interposed in front of the lens when it is desired to use a source of light situated to one side of the optical axis. The light-flint prisms rest on a separate heavy metal table and rest against one of three templates which fix the positions of minimum deviation for centres at D, H_γ, and K. The collimator and camera are carried on separate heavy double rails furnished with levelling screws. The camera lens is a Zeiss chromat of 76-mm. aperture, and the collimator lens by the same maker is of 57-mm. aperture.

Changes from one region to another can be quickly performed and the camera set to focus by a micrometer screw. The tilt of the plate is also set by a graduated arc. A number of photographs can be taken on one plate, and the numbers in brackets after the negative numbers are readings on the scale indicating the position of the plate in its vertical slides.

Scale of Photographs and Probable Error of a Single Setting (=0·00368 rev.).

Region.	K and Hγ.				D. Blue End.				D. Red End.			
	λ.	Å. per mm.	1 Micr. Divn.	p.e.	λ.	Å. per mm.	1 Micr. Divn.	p.e.	λ.	Å. per mm.	1 Micr. Divn.	p.e.
K	3600	2·41	0·012	·004	4900	8·35	0·041	·015	6200	20·65	0·103	·038
	3800	3·50	·018	·007	5100	9·81	·049	·018	6400	23·03	·115	·042
Hγ	4000	3·72	·019	·007	5300	11·48	·057	·021	6600	25·10	·126	·046
	4200	4·91	·025	·009	5500	13·30	·067	·025	6800	27·68	·138	·051
	4400	6·22	·031	·011	5700	15·25	·076	·028	7000	30·38	·152	·056
	4600	7·68	·038	·014	5900	17·33	·087	·032	7200	33·22	·166	·061
	4800	9·35	·047	·017	6100	19·51	·098	·036	7400	36·17	·181	·067

* *Astrophysical Journal*, **1**, 149 (Rowland and Tatnall).

Rowland's wave-lengths of the iron lines were used in the measures. The probable errors above are derived from twenty settings on an average good line.

My sincere thanks are gratefully tendered to Mr. J. W. Jackson, who has rendered invaluable assistance throughout the investigation; to Mr. Kriesel, who kindly presented specimens of metallic germanium; to Professor J. Smeath Thomas of Cape Town, who kindly prepared and presented the liquid chlorides of germanium and tin; and to Mr. H. Spencer Jones, H.M. Astronomer, for his kindly interest and encouragement in the work.

Royal Observatory, Cape of Good Hope:
1924 *September* 16.

TABLE II.

Spectrum of Germanium.

Vacuum Tube $GeCl_4$.	Int.	Lunt. Metallic Ge Spark.	Lunt. Int.	Lunt. Class.	Exner and Haschek. Metallic Ge Spark.	Exner and Haschek. Int.	Kobb. Corrd. to Rowland. Spark.	Remarks.
3937·9		3938·1	3	III.	3938·0	1+	..	Very diffuse.*
4085·7		4085·7$_3$	6	III.	..	..	..	
88·3		88·2$_3$	7	III.	(4088·3	4	3 Å. wide)	Two pairs of very diffuse lines.
97·9§		98·1$_3$	10	III.	(4099·9	1+	2 Å. wide)	
4100·3		4100·2$_3$	8	III.				
79·20$_3$		79·21$_4$	30	III.	4179·20	20	4178·6	
4226·77$_5$	20	4226·76$_4$	20	I.	4226·76	50	4226·1	Arc line.
60·97$_3$		60·93$_4$	20	III.	60·97	10	61·1	
1 invd.		91·80$_4$	10	III.	91·72	3+	91·6	
———		———			4602·35	1+	..	Possibly due to Li.
4686·07$_2$	10	4686·08$_3$		I.	86·09	10r	4685·4	Arc line.
4742·04$_2$	8	‡‡		I.	4744·0	3+	4743·0	Shifts to red.
..		4814·38$_4$	8	III.	———		..	Pl. 1, strip VII.
4814·88$_4$	9	‡‡		I.	..		4814·0	Shifts to red.
24·62$_3$	3	‡‡		I.				
4934·35	1	**		II.				
41·61	1	**		II.				

* Close to a nitrogen group containing three previously unrecorded enhanced nitrogen lines outside Fowler's list, at 3934·60, 3938·63, and 3942·97; also two lines at 3939·74 and 3941·03(du).

§ Not nitrogen 4097·48; the associated line at 4103·54 is very faint or absent.

‡‡ Very diffuse, not suitable for measurement.

** Diffuse.

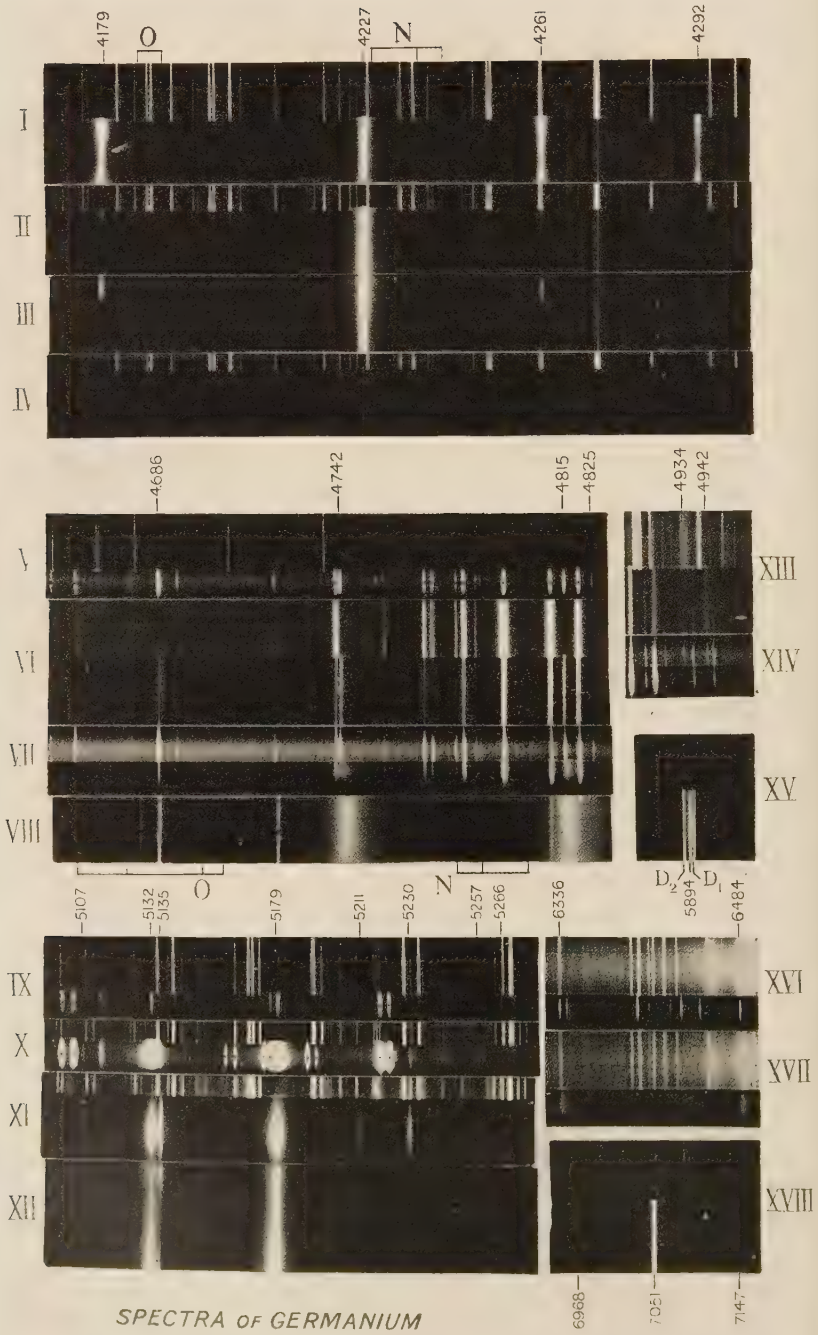
r=shaded on red side.

TABLE II.—*continued.*

Vacuum Tube $GeCl_4$.	Int.	Lunt. Metallic Ge Spark.	Int.	Class.	Exner and Haschek. Metallic Ge Spark.	Int.	Kobb. Corrd. to Rowland. Spark.	Remarks.
5106·98$_2$		5107·02$_3$	2	III.				
31·86$_4$	7	‡‡		I.	..		5131·8	
34·88$_2$		34·91$_4$	10	III.	..		34·8	
78·74$_3$	7	‡‡		I.	..		78·5	
5210·40		5210·61$_3$	2	III.	..	$\Delta\nu$ 169		
29·54		29·62$_3$	6	III.	..		5229·5	
56·76		56·72$_3$	2	III.	..		56·5	
66·09$_2$	2	Fe invd.		II.	$\Delta\lambda$			
5513·46$_2$	3	5513·84 r$_3$		II.	+·38			
64·94$_2$	5	65·32$_3$		II.	+·38			
5607·20$_2$	6	5607·15$_3$		II.				
[16·34$_2$]	5	Fe invd.		II.				Origin in doubt.
21·63$_2$	6	22·04 r$_3$		II.	+·41			
56·20$_2$	5	56·54 r$_3$		II.	+·34			
64·45$_2$ 65·08$_2$	3 2	65·27 r$_3$		II.	+·50			
92·16$_2$	6	92·69 r$_3$		II.	+·53			
5702·05$_2$	4	5702·26$_2$		II.	+·21			
18·12$_2$	1	18·53$_2$		II.	+·41			
5802·33$_2$	0	5802·75$_3$		II.	+·42			
93·64$_4$	10	93·68$_4$		I.	+·40	$\Delta\nu$ 360	5893	Between D lines.
5967·80	0	5968·36		II.	Mean		..	
6021·25$_3$	10	6021·43$_3$		I.			6021	
78·47$_2$	0	78·42$_4$		I.	$\Delta\lambda$		..	
6336·4 *	3	6337·9		I.	+1·5	$\Delta\nu$ 360	6337	Shifts to red 1·5 Å.
6484·4 †	5	6486·0		I.	+1·6		——	Shifts to red 1·6 Å.
6966·8	W	6967·7		I.		$\Delta\nu$ 360		Wave-lengths provisional. 7146 extrapolated.
7050·0	S	7051·2		I.				
7145·8	W	7147·1		I.				

Suffixes after wave-lengths denote number of measures when above one.

‡‡ Very diffuse, not suitable for measurement.
* There is a chlorine line at 6335·5.
† Strong nitrogen line at 6482·3.



SPECTRA OF GERMANIUM

DESCRIPTION OF PLATE I.

I. Neg. 227 (30). Upper—Fe comparison spectrum, Exp. 15^s. Lower—Spark between metallic terminals, 18-in. coil, hot spark, edge of bright part of vertical spark. Exp. 15^s. Enhanced lines 4179, 4261, and 4292 well shown. Iron impurity and air-lines shown.

II. Neg. 220 (20). Same exposures but with self-induction 248 μH, enhanced germanium lines suppressed except at pole, no air-lines.

III. Neg. 220 (45). Exp. 10^s, no window, spectrum of upper pole shows the enhanced lines. 4260·94 on red side of iron impurity 4260·64.

IV. Neg. 227 (40). Conditions as in I., exposure 1^s.

V. Neg. 259 (40). Upper—Fe comparison, Exp. 15^s. Lower—End-on tube of $GeCl_4$, pressure 10^{mm}, 6-in. coil, direct discharge. Exp. 5^m. The three very diffuse lines 4742, 4815, 4825 in spark spectrum (VIII.) are here sharp, Cl lines numerous. The dotted lines are 12 Cl lines recorded by Eder and Valenta, $\lambda\lambda$ 4654–4819.

VI. Neg. 290 (20). Upper—$SnCl_4$ comparison, Exp. $12^{m}\cdot5$, at beginning and end of lower exp. ($=25^m$), direct discharge, some of the chlorine lines diffuse and displaced to red. Lower—$GeCl_4$ capillary tube, Exp. 10^m, 6-in. coil, capacity 0·0025 μF, inductance 183 μH, no spark-gap, showing 4742 and 4815 almost as sharp as 4686.

VII. Neg. 259 (20). Upper—Conditions as in V., Exp. 5^m. Lower—Consecutive exposure, Exp. 8^s, capacity 0·0025 μF, no inductance, small spark-gap. 4686 remains sharp. 4742 and 4815 much diffused and displaced to red, certain Cl lines weakened or suppressed, notably 4654 and 4661, others not affected.

VIII. Neg. 227 (30). Vertical spark between Ge metal terminals, Exp. 15^s, 18-in. coil, edge of bright part of hot spark, 4742, 4815, 4825 very diffuse, 4686 sharp. Dotted lines (3) due to zinc. N and O lines faintly shown.

IX. Neg. 260 (20). Upper—Fe comparison, Exp. 90^s, at beginning and end. Lower—$GeCl_4$ end-on tube, Exp. 15^m, 6-in. coil, direct discharge. 5132 and 5179 sharp. Enhanced lines absent. 5266, Class II., present.

X. Neg. 261 (20). $GeCl_4$ end-on tube, Exp. 2^m, 6-in. coil, capacity 0·0025 μF, no inductance, small spark-gap. Enhanced germanium lines shown (5), 5266 absent, 5132 and 5179 diffuse. The chlorine lines 5099–5222 (11) recorded by Eder and Valenta are dotted. The three central pairs become enhanced, 5099 and 5103 are triple aud double respectively.

XI. Neg. 234 (30). Ge metal in hydrogen, Exp. 5^m, 6-in. coil, capacity 0·005 μF, no self-induction, small spark-gap. Brightest part of spark in centres, enhanced lines shown.

XII. Neg. 230 (50). Ge metal in air, Exp. 10^s, 18-in. coil, hot spark, no window, longer spark, brightest part at poles.

XIII. Neg. 270 (20). $GeCl_4$ capillary, Exp. 48^m, 6-in. coil, direct discharge, shows Ge pair 4934, 4942, Class II. Strong Cl pair, Eder and Valenta's 4918–4925; the latter is double.

XIV. Neg. 262 (20). Upper—$GeCl_4$, as in IX. Lower—Same tube, Exp. 30^s, with capacity 0·0025 μF, small spark-gap and no inductance, showing disappearance of Ge 4934 and 4942.

XV. Neg. 297 (20). Ge metal, Exp. 20^s, 18-in. coil, cool transverse spark, showing Ge 5894 between the two D lines. (Na impurity.)

XVI. and XVII. Negatives 262 (30 and 40). Consecutive exposures, $GeCl_4$ end-on tube. Upper—15^m Exp., direct discharge. Lower—30^s Exp., enhanced-line conditions, showing suppression of previously unrecorded Cl lines; the two red Ge lines more diffuse and markedly shifted to red. Compare VII.

XVIII. Neg. 231 (20). Upper—Cobalt comparison lines 7017, 7053, 7085. Lower—Three extreme red Ge lines, Ge metal, Exp. 3^m, 18-in. coil, hot spark.

Description of Plate 2.

I. Neg. 228 (20). Upper—Fe comparison, Exp. 3^m. Lower—Ge metal spark. Exp. 1^m, 18-in. coil, hot spark,* showing Ge 5884, 6021, and 6078. Nitrogen groups 5667–5711 and 5928–5942 present. Ge lines of Class II. absent.

II. Neg. 233 (20). Upper—Fe comparison as above. Lower—Ge metal spark. Exp. 3^m, 18-in. coil, cool spark,† showing Ge lines of Class II.

III. Neg. 262 (20). Upper—$GeCl_4$ end-on tube. Exp. $7\frac{1}{2}^m$, beginning and end ($=15^m$), direct discharge, showing Ge lines, Class II. Lower—Middle exposure 30^s, same tube, capacity 0·0025 μF, no self-induction, small spark-gap. Showing absence of Class II. lines, although 5894 and 6021 are stronger. Eder and Valenta's Cl lines 5570 (shifted to red) and 5635 persist, as does the Cl line 6095 recorded by Hagenbach and Konen. Of the two Hg lines, 5770 and 5791, only the latter remains.

IV. Neg. 270 (20). Upper—Fe (moistened with Na salt), Exp. 90^s, beginning and end $=3^m$. Lower—$GeCl_4$ capillary tube, Exp. 48^m, 6-in. coil, direct discharge.

V. Neg. 272 (20). Upper—Fe as above. Lower—Same $GeCl_4$ tube. Above exposure caused the deposition of the greater part of the Ge in the bulb (exposures of 2^m, 15^m, and 15^m followed above). Exp. 60^m, direct discharge, Ge 5894 and 6021 faint, Cl 6095 stronger, Cl lines outside region shown, strong. H_α very strong, due to liberation of occluded hydrogen (not shown). Na lines shown at one spot in capillary, lower end of line.

VI. Neg. 227 (30). Upper—Fe 15^s. Lower—Ge metal spark, edge of bright part of hot spark. Exp. 15^s, showing two pairs of diffuse Ge lines, Pb 4058, oxygen lines, nitrogen enhanced pair too faint to interfere.

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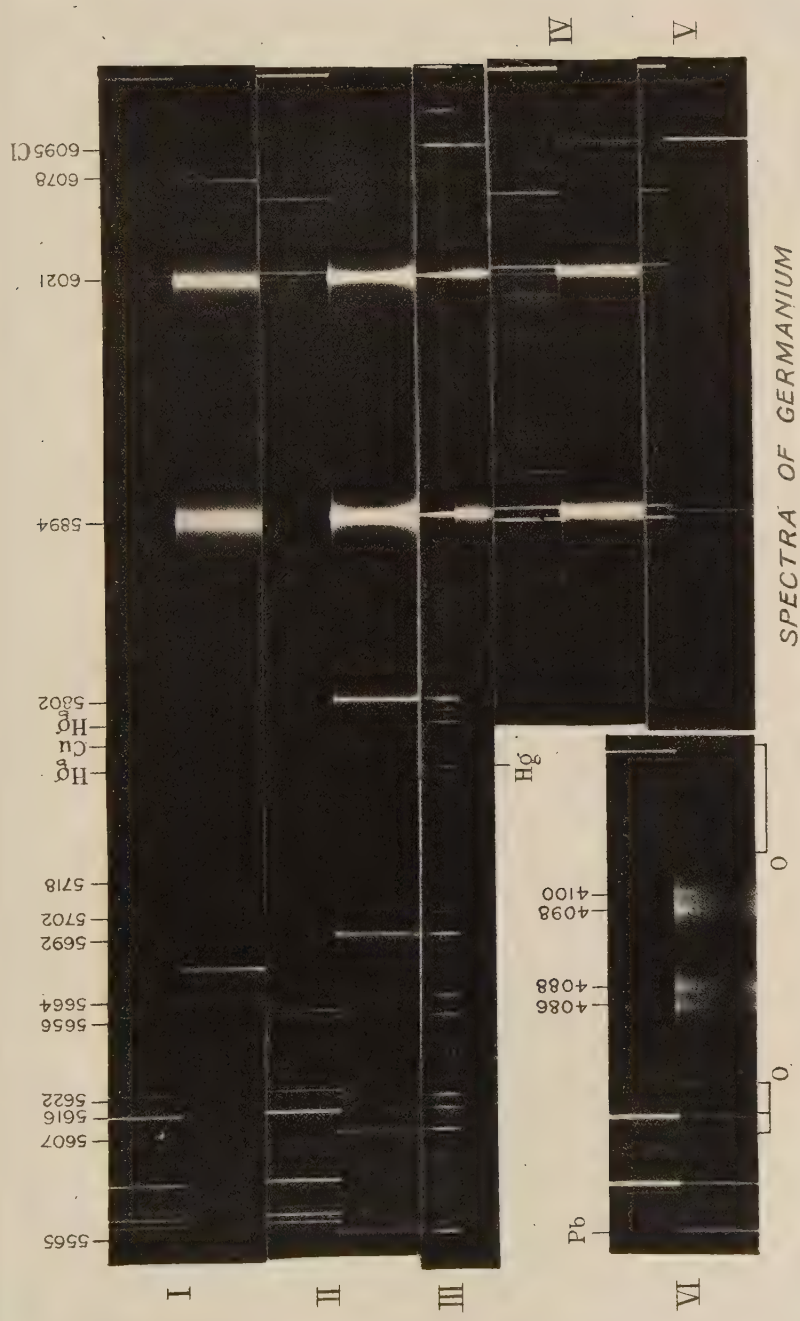
* "Hot spark" is an abbreviation for capacity 0·06 μF, no self-induction, and spark-gap in secondary circuit.

† "Cool spark" is an abbreviation for capacity 0·06 μF, inductance 248 μH, and small spark-gap.

μF and μH denote microfarads and microhenrys respectively.


SPECTRA OF GERMANIUM
I
II
III
IV
V
VI
6095 Cl
6078
6021
5894
5802
Hg
Cu
Hg
5718
5702
5692
5664
5656
5622
5616
5607
5565
Hg
4100
4098
4088
4086
O
O
Pb

Apparatus for Broadening Spectra. By William J. S. Lockyer, M.A., Ph.D. (Plate 3.)

Introduction.—Every spectroscopist knows that much valuable time is occupied in securing *broad* stellar spectra at the telescope.

In the case of the slit spectroscope the image of the star is run very slowly along the slit, this run being accomplished by placing the slit in the direction of right ascension and giving a slight rate to the driving clock. The width of the spectrum depends on the length of the run given.

In the case of the prismatic camera, in which instrument the thin edge of the prism is placed parallel to the direction of right ascension, the driving clock is also given a rate, but the spectrum is made to trail across the photographic plate, at right angles to the direction of dispersion of the prism, until the required width of spectrum is secured.

In both forms of instruments great width means considerably long exposures at the telescope, and therefore a reduction in the number of negatives that can be obtained during any one night. For faint objects, which normally require a long exposure, broad spectra are out of the question.

For measurements of the wave-lengths of the lines with a micrometer, the width of the original spectrum need only be sufficiently broad to identify the lines, but in some spectra, such as those which exhibit both bright and dark lines or flutings, a greater width secured at the telescope is preferable, and even necessary, to distinguish between the bright lines and interspaces between dark lines.

Now, unless the spectra obtained are very broad they are practically useless for satisfactory enlargement for reproduction purposes, because no matter how much the spectrum may be enlarged the width still remains very small relatively to the length.

Direct enlargements of stellar spectra are unsatisfactory for three definite reasons, namely :—

1. Grain of the sensitive emulsion.
2. Irregularities due to the driving clock.
3. Irregularities due to intermittent clouds.

The enlargement of the grains of the plate gives rise to a mottling over the whole spectrum, and this tends also to obliterate the presence of the faint lines in the spectrum.

It is the peculiarity of negatives taken with a prismatic camera that they exhibit irregularities resembling the lines along a spectrum, observed when dust particles lie on the jaws of the slit of a spectroscope. These irregularities are chiefly due to the time of revolution of the main driving screw of the telescope, and are most pronounced in the case of bright stars. The enlargement of such negatives emphasises the conspicuousness of these lines and detracts from the beauty of the spectrum.

Again, during an exposure with the telescope, intermittent clouds give rise to breaks in the width of the spectrum not unlike the irregularities due to dust particles on the jaws of a slit spectroscope. These also on subsequent enlargement spoil the appearance of the spectrum.

Any means, therefore, by which greatly enlarged and widened spectra can be obtained from the original negatives, without exhibiting any of the above-mentioned deficiencies, is most desirable.

Previous Forms of Apparatus.—Numerous devices for widening spectra have been employed from time to time, and the following brief accounts give some idea of the chief principles involved.

Professor E. C. Pickering* broadened all his spectra by means of a cylindrical lens, but chiefly with the object of eliminating the mottling due to the enlargement of the grains of the emulsion. The cylindrical lens made the grains entirely disappear, since they were drawn out into numerous fine lines, the length of which equalled the width of the spectrum. Their entirely accidental distribution over the whole negative made the resulting intensity of the spectrum quite uniform.

Lockyer† at a later date employed a framework for carrying the negative, which could be oscillated up and down in a vertical direction between four guide-wheels. The upper portion of the framework was attached to a long lever (pivoted near its centre) by a short arm, and the other end of the lever, which carried a counterweight, was also connected by a short arm to a point on the periphery of a wheel driven by clockwork.

The resulting motion of the framework was therefore variable throughout, the maximum and minimum velocities occurring in the middle and ends of the width of the run. The original negative was placed in the negative-holder and the enlargement made on a fixed plate in the camera.

Scheiner‡ improved on this apparatus later, and gives a description and illustration in his treatise on *Astronomical Spectroscopy*. The instrument was made on the same principle as that of Lockyer, only he caused the plate on which the enlargement was to be made to undergo the oscillatory motion instead of the original negative.

The frame which carried the photographic plate ran on four wheels between two vertical guides and rested on the periphery of an eccentric. The weight of the plate-holder was reduced by counterpoise weights attached to cords passing over two pulley wheels above the plate-holder. The form of the eccentric was that of two opposing spirals, so that when the eccentric was revolved an up-and-down motion with uniform velocity was obtained. The eccentric was actuated by clockwork which was governed by a spring pendulum. At the turning-points where the motion was reversed, a slit of slightly less width than the whole extent of the motion was inserted in front of the photographic plate to shut off the light.

Another method that has been employed involved the use of a pendulum. A fixed enlarged image of a spectrum was thrown on to a photographic plate placed at right angles to the beam and situated at the lower end of a long pendulum in the plane of its swing: the direction of dispersion was set in the vertical position. The exposure was made during the oscillations of the pendulum, thus broadening the spectrum

* *Harvard Coll. Ann.*, **26**. part 1, p. xix, 1891.
† *Phil. Trans. Roy. Soc.*, **184**, A, p. 684, 1893.
‡ *Die Spectralanalyse der Gestirne*, p. 103, 1890.

to the extent limited only by the size of the plate and the amplitude of the pendulum's swing. Naturally, a long pendulum was necessary to reduce the curvature of the broadened lines.

While all these methods eliminated the various deficiencies previously enumerated, there seemed ample room for a more improved and simple apparatus.

The Clepsydra Principle.—The apparatus which forms the subject of the present paper arose out of an attempt to improve the instrument used by Sir Norman Lockyer at South Kensington.

From the experience gained by the writer in 1904 in the use of the spectroheliograph mounted at the Solar Physics Observatory at South Kensington, in which instrument a homogeneous horizontal motion was secured by the flow of oil in a cylinder, it was considered that a motion in a vertical direction could be obtained by employing the same principle. Instead of the negative being oscillated in a vertical direction, the idea was to simply allow the negative to fall by gravity, the necessary speed being governed by the flow of oil.

All the earlier methods could be used by illuminating the negative with either a variable or constant light source, but the present principle necessitates a constant light source throughout the whole time of an exposure. Reference to this point will be made further on.

Satisfied that the clepsydra principle was the correct line to work on, a sketch was made embodying this principle and giving the necessary refinements and adjustments required for satisfactory operation. Messrs. Cooke, Troughton & Simms undertook the construction of the apparatus, and the experience so far gained shows that it works with exceeding smoothness.

Before proceeding to the description of the instrument, reference should first be made to a piece of apparatus for broadening spectra designed and used by Ralph H. Curtiss at the Detroit Observatory of the University of Michigan.

Curtiss* describes this instrument very briefly and gives no illustration of it, probably because he considered it only as an experimental instrument to be improved later. His account shows, however, that he has adopted the same principle of oil flow and carried out the design in a very similar manner to that about to be described. As this account was unknown to the writer until after the Sidmouth instrument had been received from the makers, it is remarkable that they should so closely resemble each other. It is true, as Curtiss states, that his apparatus is "avowedly in preliminary form," no anti-friction bearings and only a cylinder of ordinary tubing which had not been bored being used, but judging by the very fine results he has already published, its performance is extremely satisfactory.

In spite of the above, it is thought that a description of this simple instrument, and the experience gained with it, may prove useful to spectroscopists who have to deal with stellar and nebular spectra. Further, it is hoped that the more general use of this instrument will enable a greater number of spectra to be reproduced and published: up to the

* *Pub. Astr. Obs. Univ. Michigan*, 2, 182, 1916.

present their appearance has been very restricted owing, no doubt, to the unsatisfactory nature of the ordinary enlargements.

Description of the Apparatus.—The accompanying illustration (Plate 3, fig. 1) gives a general view of the instrument. (Scale : 0° to 120° on the thermometer scale is equal to 5·5 inches.) The portion of the frame (A) is one end of a standard enlarging camera. Sliding horizontally in this end, between slots, is a stout wooden board (B). Mounted in front of this board, at a distance of 1·75 inches, is a brass frame (C) with a horizontal slot cut out of the middle (not seen in plate) measuring 5 × 1 inches. This frame (C), 7·5 inches broad in the middle, has fixed to its lower end a piston-rod (E) which passes into the vertical cylinder (F) and has attached to its lower extremity a heavy piston.

The frame (C) can thus only move in a vertical direction, and its movement is restricted to one plane only, by the two anti-friction wheels (D), the rolling surfaces of which are cut V-shaped so as to guide the two specially trued V-cut running edges of the frame (C). The wheels (D) stand out about 1·75 inches from the board (B), and their axes are fitted into brass plates capable of adjustment horizontally in fixed grooves mounted on the board (B). This is for the purpose of adjusting the wheels accurately against the guides on the sides of the plate (C).

The space between the frame (C) and the board (B) is rendered light-tight by a small mahogany frame fixed to the board (B) and nearly but not quite touching the back of the brass frame (C).

The insides of the upper and lower extremities of the cylinder (F) are connected externally by means of two tubes. That on the left (G) is fitted with a stop-cock (H) and is arranged to allow a *quick* flow of oil from the upper to the lower side of the piston, when the plate-holder (C) is being raised. The tube on the right is encased in the portion of the casting (K) and is fitted with an adjustable valve, operated by the graduated micrometer head (L), to allow a *slow* flow of oil to pass from the lower to the upper part of the piston as the frame (C) with piston-rod and piston fall by gravity.

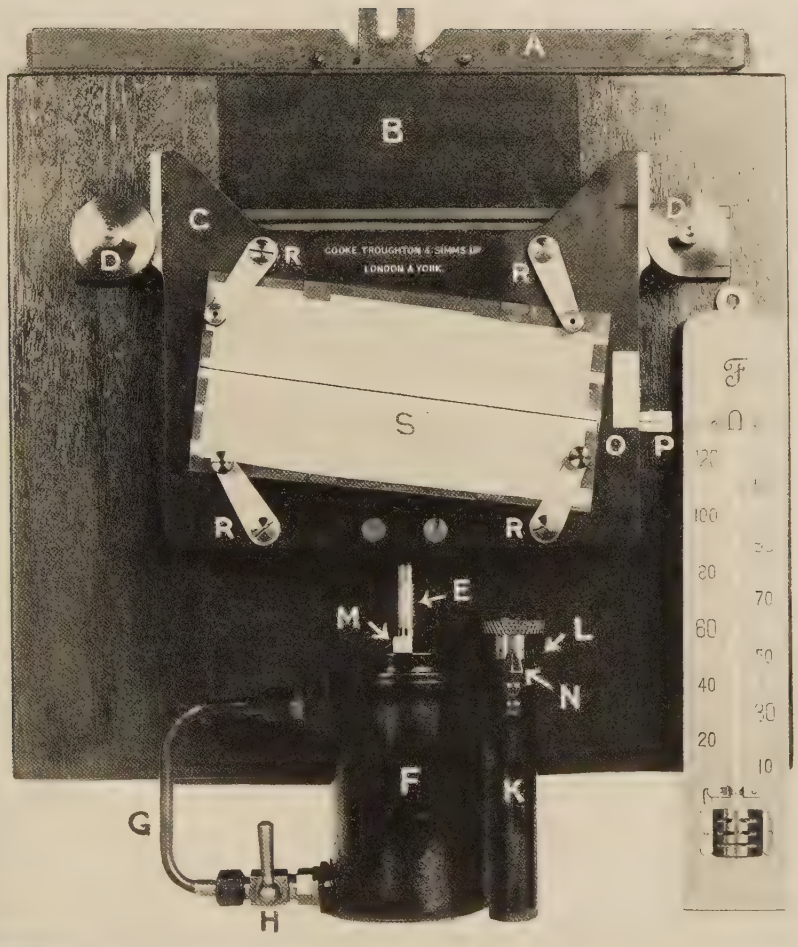
The cylinder and the two external connections are filled with fine-grade clock oil, through an opening (M) on the top of the cylinder, care being taken to ensure that the whole of the interior is devoid of air bubbles. This is easily secured by working the frame (C) up and down, when the filling process is nearly completed, until no air bubbles are seen to issue through the orifice (M).

The micrometer head (L) is graduated into fifty divisions, and the readings are made at a fixed pointer (N). On the right-hand side of the frame (C) is a scale (O) graduated in twentieths of an inch to indicate the distance through which the frame falls, as determined by the fixed mark (P) connected rigidly with the mahogany board (B).

In front of the brass frame (C) are four brass clips (R) to hold the negative in position, in front of the horizontal slot in the movable frame (C).

The operation of working the instrument is as follows :—

First : open the stop-cock (H) (it is closed in the illustration) and raise by hand the frame (C) until the piston is at the top of the cylinder or the zero of the scale (O) is opposite the pointer (P). Then close the


A
B
C
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D
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R
R
R
COOKE, TROUGHTON & SIMMS LP
LONDON & YORK.
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20
10

FIG. 1.

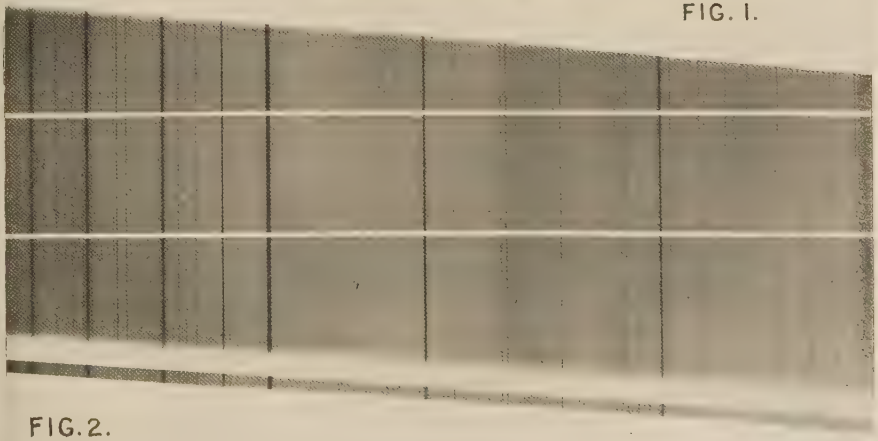


FIG. 2.

stop-cock (H), and seeing that the micrometer valve is also closed, *i.e.* reading O, then the frame (C) will remain in that position without assistance.

When the desired speed of the frame (C) is known, the micrometer screw-head (L) is then set to the requisite reading, and the frame will fall slowly down at the required rate.

Preparation of the Negative.—The preparation of the negative to be widened depends considerably on the negative itself. Particulars to be considered are whether the spectrum is wide or very narrow; whether the spectral lines are at right angles or oblique to the direction of dispersion, or even have a small curvature; and whether the spectrum is broken up in width by intermittent clouds.

In all cases, however, the whole negative must be made opaque to light with the exception of the strip along the spectrum which has been selected for broadening. The method adopted to ensure this is to use thin sheets of tin-foil, cut with a sharp razor into strips about three-quarters of an inch broad and in length equal to the length of the plate. Each negative requires two strips, and each strip has fixed to its extremities a short piece of lantern-slide binding strip projecting about half an inch.

These strips are then placed on the film side of the negative and brought sufficiently near together and parallel to each other, to include between them the portion of the spectrum to be widened. When in position the free ends of the lantern binding strips are turned round the edges of the plate and stuck to the back of the plate. This method ensures that no damage is done to any part of the film surface of the original negative, a very necessary precaution.

Taking great care to remove any particles of dust that may be present on the strip of the spectrum to be broadened, a clean glass plate equal in size to the negative, is then placed on the film side of the negative, and the two plates bound up together. The whole plate is then made light-tight by pasting any kind of opaque paper on the outside of the cover-glass.

Such a negative (S) prepared in this manner, in a position for broadening, is illustrated in the accompanying Plate 3, fig. 1.

It is advisable when preparing a negative to keep the space between the pieces of tin-foil as wide as possible, as by this means the grains of the film are more completely eliminated. Further, the closer the pieces of tin-foil are placed, the greater must be the accuracy in seeing that their edges are truly clean cut, otherwise any raggedness tends to produce artificial lines in the resulting spectrum.

Adjustment and Broadening of the Negative.—The prepared negative is now placed on the movable frame (C) and fixed in position by means of the four clamps (R). The enlarged spectrum is then brought into focus on the ground glass of the enlarging camera. The first step is to ensure that the lines in the spectrum are strictly parallel to the direction of motion of the brass frame (C) carrying the negative. As this frame (C) is adjusted to move vertically, the spectral lines on the screen should also move vertically. By ruling a few vertical lines on the ground glass of the enlarging camera, it is then an easy matter to twist the negative in the movable frame (C) so that the spectral lines become parallel to the

pencilled lines. In some negatives a considerable amount of twist of the negative is necessary, and for this reason the slot in the middle of the brass plate (C) is made broader than seems at first sight necessary.

To produce the positive, from this negative, the brass frame (C) is pushed up to its highest point with stop-cock (H) open, and micrometer valve (L) shut, and then the stop-cock (H) is closed.

In this position an ordinary exposure is then made, giving a simple enlarged photograph of the spectrum without any motion. When it has been decided what speed shall be given for the broadened enlargement, then the micrometer valve head (L) is set accordingly, and the brass frame (C) allowed to fall. After it has fallen about one-tenth of an inch, the exposure is commenced and only stopped when the pointer (P) reads one inch on the scale (O).

The resulting positive thus shows the enlarged, widened spectrum, alongside a narrow, but equally enlarged, spectrum. From an examination of the latter it can be easily seen whether any defects such as dust-marks, irregularities in the cut edges of the tin-foil, etc., are present, which may cause artificial lines in the broadened spectrum.

Should such occur, then it is generally advisable to prepare the negative afresh, and repeat the whole operation, taking care to eliminate the known sources of irregularities.

It has been found best to use slow plates, and those actually employed have been Imperial Process Plates (H and D 30).

It should not be forgotten that even if all the original negatives are on the same scale, the enlargements, by the above process, will produce positives on different scales. This must be so, because the various degrees of twist given to the original negatives, in order to make the spectral lines vertical in the enlarging camera, alter the horizontal scale.

This fact is, however, of minor importance, because these positives have to be converted into negatives, and during the process they can be enlarged to any size and brought to any desired scale.

The Exposure Times.—It has been stated in a previous paragraph that it is necessary to work with a constant light source during the time occupied in making any one positive with the clepsydra apparatus. Curtiss,* in the description of his method, employed a pair of nitrogen tungsten lamps with a combined candle-power of about four hundred, using two parallel ground-glass plates to diffuse this light properly, before it reached the subject to be enlarged.

To produce a constant light, the writer made a box containing two 100-watt Mazda lamps giving a candle-power of 200, and fitted with a window, in front of which any desired number of sheets of ground glass could be placed.

Owing to the favourable position of the Norman Lockyer Observatory, which is situated 560 feet above sea-level, an attempt was first made to use daylight with the enlarging apparatus. A large plane sheet of looking-glass was placed at an angle of 45° outside the enlarging-room, about three and a half feet from the negative, to reflect zenith light directly into the enlarging apparatus.

* *Pub. Astr. Obs. Univ. Michigan*, **2**, 182.

It was found that by working on days when either the sky was free from clouds or was homogeneously covered with clouds (and even raining), these conditions were quite suitable, and at the same time the exposure times would be very much shorter than with the artificial light. The results have been so satisfactory that the light box mentioned above has never been brought into use.

The exposure time for making a positive depends on conditions other than that of the light source used. The first is the breadth of the spectrum as given by the distance between the strips of tin-foil: the larger this distance the less the exposure required. Density of the negative is also another factor, the exposure being prolonged with increase of density. Lastly, the amount of enlargement has also to be taken into account; the greater the scale required, the longer the exposure.

The length of exposure is controlled by the micrometer head (L) which operates the valve, and this determines the speed of fall of the negative attached to the brass frame (C), by the time taken to fall through every tenth of an inch.

The diagram (p. 54) illustrates three curves showing the time of fall in seconds per one-tenth of an inch run (ordinate) for every fifth division of the valve micrometer head (abscissa). The latter reads "zero" when completely closed, and one revolution of the head equals 50 divisions: the valve readings above 50 correspond to the divisions for the second revolution and are denoted by 55, 60, 65, etc.

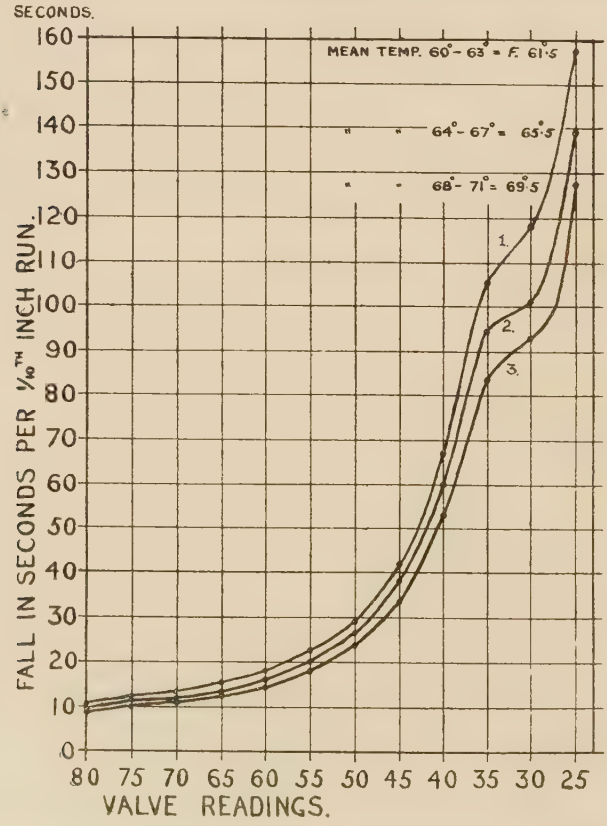
The runs were made systematically for every fifth division from 25 to 80 during the month of July. During this period the temperature, indicated by a thermometer placed alongside the apparatus, only varied from 60° F. to 71° F. at the times the readings were taken, and this explains the restricted temperatures in the figure. Every series of runs occupied about 32 minutes, and as 26 runs were taken the total time occupied was about 14 hours.

The observations were divided into three groups of temperature, namely, 60° to 63°, 64° to 67°, and 68° to 71° F., and the means of each group were formed and plotted as shown in the curves 1, 2, and 3 respectively in the figure. It will be seen that the three curves are very similar and show that as the temperature increases the rate of fall also increases, as was to be expected. A conspicuous feature of the curves is the change of curvature between the valve readings at 35 and 25, indicating a small mechanical inaccuracy in possibly the valve. The temperature differences are most pronounced when the runs are very slow but virtually disappear on very fast runs. Thus at the valve reading 35 there is a difference of 21·5 seconds for the temperature change from 69°·5 F. to 61°·5 F., a reduction of 8°·0 F. At valve reading 80, there is only a difference of 1·5 seconds for the same temperature change.

As a matter of experience, the valve readings usually employed have been between 45 and 60, so that the portion of the curve involved includes neither any large differences due to temperature nor any abrupt change of curvature as exhibited between the valve readings 35 to 25. A greater error is involved in determining the length of an exposure for a run of one inch than in any allowance made for the difference of speed

of fall due to the oil temperature. Further, a little too rapid or too slow a fall can generally be manipulated by a shorter or more lengthened development of the plate.

Production of Negative and Print.—The positive is enlarged in the ordinary way, the clepsydra apparatus having been removed and the plate-carrier inserted in its place. The positive is then placed in the carrier and focused on the ground glass to the enlargement desired, it



being remembered that the resulting spectrum (negative) to be used for the final print is a long strip cut out at right angles to the lines in the spectrum.

In order to maintain the same scale of enlargements when other positives are enlarged, it has been found of great use to make a permanent glass scale. This was accomplished by placing on the ground glass of the enlarging camera a piece of clean glass and then marking on it the positions of each of the hydrogen lines in the spectrum. By scratching lines with a diamond through those marks parallel to each other, and at right angles to the direction of dispersion and to the long side of the

glass plate, this graduated glass plate can be placed on the ground glass and used as a scale for any subsequent negative to be made.

For the production of the negative, similar process plates were again used with advantage.

When the paper print has been finally made from this negative, care has to be taken to cut out a strip so that the spectral lines are exactly at right angles to the direction of dispersion. This can now be easily accomplished by the use of the graduated glass plate mentioned above, placing the plate over the print so that the diamond-cut lines coincide with the hydrogen lines: the print may then be cut along the edge of this plate.

The accompanying illustration (Plate 3, fig. 2) of a portion of the spectrum of α Cygni is a reproduction (natural size) of a direct print from the final negative made from the enlarged positive by the clepsydra apparatus. The narrow strip at the bottom is the simple enlargement of the spectrum when the clepsydra apparatus was at rest.

The obliqueness of the length of the spectrum in relation to the lines in the spectrum is caused by placing the original negative in the clepsydra apparatus at such an angle that the spectral lines in the narrow strip are vertical or parallel to the direction of motion of the negative carrier.

The space between the two white lines, drawn at right angles to the spectral lines, represents the portion of the spectrum to be cut out and used for the purpose of reproduction. This width can be varied at will, but, as will be seen, there is a maximum limit.

Specimens of five spectra in their completed form were published * during the present year in my paper entitled "Spectroscopic and Magnitude Observations of o (Mira) Ceti, 1924."

Summary.

1. The paper explains in the first instance the object of widening stellar and nebular spectra by mechanical means for the purposes of reproduction.

2. Brief accounts are then given of various forms of apparatus which have been previously employed.

3. A new form of instrument, based on the "clepsydra" principle, is next described in detail, showing how the negative is made to fall vertically by gravity and its speed controlled by the flow of oil in a cylinder.

4. Then follows an account of the preparation and adjustment of the original negative to produce the enlarged broadened positive.

5. The results of an examination, relative to the speed of fall of the negative to determine the durations of the exposures, are next given, and curves are included which illustrate those speeds for different valve readings and for different temperatures.

6. Finally, a description is given showing how this enlarged and broadened positive is converted into a negative of any desired scale, and this is followed by the method of preparing the paper print for reproduction purposes.

* *M.N.R.A.S.*, **84**, Plates 14, 15.

Scattering and Absorption in the Atmospheres of Emission-Line Stars.
By Martin C. Johnson.

1. The recent advances in astrophysics made possible by the Saha theory have had to leave untouched the problems of those stars which have emission-line spectra. The only clue to the physics of the atmosphere of a bright-line star is due to Schuster *: from the general laws of radiation he suggested that for a star to show emission lines instead of absorption lines an atmosphere is required in which the scattering of radiation predominates over absorption, and in which the gradient of radiation intensity is slight. The present paper is an attempt, firstly, to express these requirements in terms of physical quantities whose value and variation can be found, in particular to examine the relation of scattering coefficients to various absorption coefficients and the conditions of effectiveness of each; and, secondly, to inquire whether the otherwise known physical properties of emission-line stars allow us to predict conditions in which Schuster's requirements might be expected to be fulfilled.

2. The physical meaning of Schuster's first requirement is as follows: In an absorption-line star the continuous photospheric spectrum comes through to us comparatively undimmed in the visual region by any important process except for the strong selective absorption of the reversing layer: by reason of this we cannot in general see the emission of the chromosphere. In an emission-line star, on the other hand, the photospheric spectrum is, according to Schuster, so strongly dimmed by non-selective scattering that there is not enough reversing layer absorption to compete with chromospheric emission. Now the chief difficulty in giving quantitative values to the absorption and scattering coefficients is that the recent researches on which our knowledge of these coefficients is based do not generally deal with the same phenomenon. Let us define K as the "opacity" of a gas of thickness z, on entering and leaving which the intensity of a beam of radiation is I_0 and I respectively. This will then be the most general coefficient of the obstruction that an atmosphere can offer to radiation, and

$$I = I_0 e^{-Kz}.$$

But the opacity is made up of several types of absorption and scattering: changed to mass coefficients by means of the density ρ,

$$\frac{K}{\rho} = \frac{\kappa_1}{\rho} + \frac{\kappa_2}{\rho} + \ldots + \frac{s_1}{\rho} + \frac{s_2}{\rho} + \ldots,$$

where κ_1, κ_2 refer to absorption by different processes, and s_1, s_2 refer to scattering by different kinds of particles.

Now a great deal of the recent advance in our knowledge of absorption has been concerned with K, but for the present problem it is the separate values of κ_1, s_1, etc., which are required; those coefficients which have been obtained from radiative theories of stellar equilibrium must

* *Ap. J.*, 1905 Jan.

be subdivided and investigated as covering several separate mechanisms, since the absorption involved in radiation pressure is the opacity and makes no distinction between the different kinds of scattering and the different ways in which absorption may occur. For the present purpose the most convenient way of making the discrimination between the coefficients will perhaps be to classify as follows :—

(*a*) Scattering by free electrons, by bound electrons, by molecules.
(*b*) Quantised line absorption by atoms, and non-quantised continuous absorption by atoms.
(*c*) Thermal transformation of radiation.

The data available concerning all these processes are to be discussed critically for application to Schuster's criterion for bright-line emission.

3. Discussions of the scattering of radiation are either based on J. J. Thomson's equation or on Rayleigh's equation : as in the present case all possible causes of scattering have to be compared, it will be worth while very briefly to indicate the reasons why one or the other can be used in different cases.

The Thomson coefficient,

$$s = \frac{8\pi}{3}\frac{e^4}{m^2c^4}\mathrm{N},$$

is based on the reaction of the charge e of an electron of mass m with the electric vector of the radiation whose velocity of propagation is c. The scattered energy is proportional to N, the number of scattering centres per unit volume. Now the equation is generally associated with the particular case of X-ray scattering by bound electrons, in which case

$$\mathrm{N} = n\mathrm{Z},$$

where Z is the number of electrons per atom and n the number of atoms per unit volume. Numerical values will be inserted after comparing with the Rayleigh equation the possibilities of applying the Thomson equation outside that particular case of X-rays and bound electrons. The Rayleigh coefficient is

$$s = \frac{8\pi^3}{3}\frac{q^2v^2}{\lambda^4}\mathrm{N},$$

where q is a function of the refractive index of the medium and v is the volume of a scattering particle which must be small compared with the wave-length λ.*

* Contradictions in the Rayleigh equation are sometimes implied owing to the confusion of alternative forms in which N appears in numerator and denominator. The more usual form is

$$s = \frac{32\pi^3(\mu-1)^2}{3\mathrm{N}\lambda^4},$$

where μ is the refractive index of the medium. This is not to be interpreted as meaning the scattered energy is inversely proportional to the number of centres per unit volume, as the place of N here is due to the expression of q^2 as a function of $1/\mathrm{N}^2$. See derivation of the formula in Schuster's *Optics*.

This equation is usually associated with scattering by particles of molecular size, and its usefulness for this purpose is confirmed by the fairly correct values of the number of molecules per unit volume which can be obtained from it. Its extension to electron scattering is explained by Sommerfeld* as follows: Both the Thomson and the Rayleigh equations can be deduced from the classical theory of the radiation from an electron; the former results if the energies from separate electrons are simply superposed, which is the case either with free electrons or with bound electrons when the wave-length of the radiation is small and of X-ray order. The latter, or Rayleigh equation, results if not energies but amplitudes are superposed, which is necessary in the case of bound electrons and wave-lengths of optical order.

Now, although in the body of a star radiation is strongest in the X-ray region, for the layers above the photosphere which are concerned in the Schuster theory we only deal with the visible spectrum. According to the above considerations from Sommerfeld, therefore, the scattering by free electrons must be calculated by the Thomson equation, but molecular and also bound-electron scattering by the Rayleigh equation.

Let s_1 be the free-electron scattering coefficient, s_2 that for molecules, s_3 for bound electrons. Inserting numerical values, we have for the Thomson equation

$$s = 6 \times 10^{-25} N.$$

Now Barkla's experiments find for the scattering of X-rays by air at atmospheric pressure

$$s = \cdot 00025,$$
$$\therefore \quad s/\rho = \cdot 2,$$

combining the experimental results with the electronic constants

$$N = 4 \times 10^{20}.$$

But N_0, the number of molecules per c.c. at atmospheric pressure and 0° C., is $\cdot 27 \times 10^{20}$. Whence for X-ray scattering by air there are about 14 centres of scattering per molecule. On the above theory from Sommerfeld this equation can only be transferred to the region of optical wave-lengths in a stellar atmosphere for the case of free electrons: in a completely un-ionised gas the scattering by this process would therefore be nil. In a hypothetical case in which every atom was once ionised we would have

$$N = N_0$$

and the scattering would be equal to $\frac{1}{14}$ of the bound-electron X-ray scattering under the same conditions. In general, then,

$$s_1 = 1 \cdot 8 \times 10^{-5} N/N_0$$
$$s_1/\rho = 1 \cdot 4 \times 10^{-2} N/N_0.\dagger$$

* *Atomic Structure*, pp. 25 and 533.

† The same numerical value could be deduced from an equation of Stewart in *Phys. Rev.*, 1923 Oct., p. 328, but the present treatment, based on experiment, illustrates the conditions under which each equation becomes valid.

For s_2 Rayleigh obtained numerical values in *Phil. Mag.*, **47**, 377, by his equation: he found the thickness of air at normal temperature and pressure required to reduce the intensity of λ6000 to $1/e$ of its value to be 83 kilometres. Replacing his value of N_0 by a more recent determination, the distance is found to be 120 kilometres. This implies

$$s_2 = \cdot 8 \times 10^{-7}$$
$$s_2/\rho = \cdot 6 \times 10^{-4}.$$

Now for bound-electron scattering on the Rayleigh equation (s_3) Stewart and Russell * give an equation for the distance in kilometres required for $1/e$ reduction of intensity of λ6000 at 5000° abs. temperature

$$x = \frac{2100}{y^2 p},$$

where y is the ratio of the refractive index of the gas to that for air at the same density, and p is the pressure in atmospheres. Now, reduction of this to 0° C. results in the same numerical coefficient as found for s_2. Stewart and Russell's derivation is not given in the paper, but if valid it means that each molecule acts as one scattering centre with respect to its bound electrons; *i.e.* s_3 reduces to s_2 and is independent of the kind of element or degree of ionisation, unlike X-ray scattering. Thus ionisation increases s_1, but otherwise does not affect the scattering.

Now the above calculation of scattering coefficients for free electrons, bound electrons, and molecules ignores the question of selective scattering except in so far as the wave-length enters into the Rayleigh equation; *i.e.* it ignores resonance scattering, since the Rayleigh law assumes there is no coincidence of radiation frequency with any free period in the atom. Stewart, who has modified the Rayleigh law to account for this, states † that a bound electron scatters less energy than a free electron from radiation of frequency less than ·71 of its natural frequency, but more than twice as much for radiation in frequency range ·71 to 1·85 times its natural frequency. The relation of scattering processes to atomic frequencies has not been traced as clearly as the relation of absorption processes to energy levels and to molecular rotation frequencies has been by the quantum theory, and apart from the latter it is not very clear what is meant by resonance; especially is this true for monatomic gases which mainly constitute stellar atmospheres. However, Stewart's introduction of a small multiplication factor in s_3 scattering would at most result in this coefficient becoming comparable to s_1 in magnitude in the case of low ionisation: in the highly ionised outer layers of the hotter stars, however, N/N_0 would be a large fraction, while a very small value of N/N_0 would be needed even for resonance to make s_3 comparable with s_1.

4. Consider now what share the κ coefficients have in the opacity. It has become possible recently through Einstein's theory of the thermodynamic equilibrium between radiation and matter to calculate true

* *Ap. J.*, 1924 May, p. 199. † *Ap. J.*, 1924 Jan.; *loc. cit.*, 1924 May.

absorption coefficients; *i.e.* the atomic constant concerning the electron transitions between orbits whose energy difference is that of the quantum absorbed. There are several uses of this method to be distinguished according as valency electrons and optical frequencies or deep-seated electrons and X-ray frequencies are considered, and according as the electron transitions are from the orbit of an atomic electron to that of a free electron or vice versa, or from one free to another free orbit through the atomic structure. The former transitions, having definite energy levels as initial or final states, are responsible for line spectra; the latter, being concerned only with hyperbolic paths which are not quantised, are responsible for continuous absorption and emission.

Kramers * has used the method for hyperbolic-hyperbolic transitions for X-ray frequencies, and Eddington has shown † that when applied to the material of the body of a star this type of absorption, together with that provided by hyperbolic-elliptic transitions (*i.e.* capture of free electrons and, conversely, ionisation), is less than enough to account for the opacity he had deduced from the radiative theory of equilibrium in the case of a particular star. In the notation of the present paper

$$\kappa_1 + \kappa_2 = \tfrac{1}{8}\mathrm{K},$$

where κ_1 is the mass-absorption coefficient corresponding to the hyperbolic-hyperbolic transitions and κ_2 that corresponding to capture and ionisation, while K itself is of the order of 40.

Now the scattering and absorption coefficients so far considered are not coefficients for a homogeneous material acted on by a single frequency: the scattering coefficients refer to any possible assemblage of gases and radiations, while the Eddington coefficients assume several separate elements in turn but are not monochromatic. These must be distinguished from the next type of coefficient. Milne has used the Einstein method to calculate the transitions of valency electrons resulting in continuous absorption beyond the limit of an optical series, but he has given reasons ‡ for considering that it also gives the order of magnitude of the monochromatic line absorptions within optical series. Milne finds

$$\kappa_{1\lambda} = 10^8 \text{ to } 10^9$$

where $\kappa_{1\lambda}$ is the mass-absorption coefficient for a single frequency corresponding to electron capture. Now from a calculation of the equilibrium of Ca^+ under radiation pressure in the solar chromosphere,§ Milne finds the total opacity is of the order of 8×10^9; also experimental data due to R. W. Wood give a monochromatic opacity for Hg of the order 10^9 so long as the absorbing material is the same as that of the source of radiation. Hence from both these sources of evidence

$$\kappa_{1\lambda} \fallingdotseq \mathrm{K}_\lambda.$$

That is to say, the monochromatic coefficient for electron capture and ionisation is approximately as large as the monochromatic total opacity.

* *Phil. Mag.*, 1923 Nov.
† *M.N.*, **84**, 104.
‡ *Phil. Mag.*, 1924 Jan., p. 218.
§ *M.N.*, **84**, 354.

It must be kept in mind that while the general stellar material will contain only a very small proportion of atoms in a condition to absorb any particular frequency, the high-level chromosphere is made up almost entirely of a homogeneous gas: this concentration difference might go far to close the gap between the Eddington general coefficient and the Milne monochromatic coefficient.

5. The fact that in Milne's theory, though not in Eddington's, the quantum true absorption coefficient is about as great as the total opacity, raises the question of the place of the absorption process of thermal transformation of radiation, *i.e.* the reappearance of energy lost in the opacity as random kinetic energy of molecules. This obscure process has not been cleared up by the recent researches which have provided the data we have just been discussing, but it cannot be passed over here, since a recent writer* has provided a theory of thermal absorption by free electrons which he suggests may be the principal source of the opacity of an ionised gas. He seems to take the maximum classical energy relation between the electric vector in the radiation and the charge of an electron, and assumes that the whole of the energy represented by the vector can be diverted from the radiation by the electrons and transformed to kinetic energy, the only variable quantity being the number of collisions by which the energy already diverted is passed on to atoms. But the mechanism by which an electron could divert the energy of radiation is completely unknown apart from scattering except in the case of the radiative alterations in the elliptic or hyperbolic paths of an electron discussed above in the quantum coefficients. In fact, quantitatively the hypothetical mechanism of Stewart has to depend on the very part of electron theory which is physically least established, that of the share of the conductivity electrons in the thermal properties of metals. For the present purpose of comparison with known processes of opacity we will first see what can be obtained from the relatively more clearly understood case of quantum absorption: since Milne's $\kappa_{1\lambda}$ seems to constitute most of $K\tilde{\lambda}$, it seems likely that it may include thermal transformation itself. Consider the possible results of the transformation of the energy of radiation to heat. It may reappear either as translational or as rotational energy. The latter is at present the more satisfactorily calculable of the two, Bjerrum's theory of the energy absorbed in the rotational degrees of freedom of molecules providing a fairly satisfactory quantitative account of band spectra. These, however, are referred entirely to polyatomic molecules, while in astrophysical theory only the coolest stars allow any compounds to be stable: for thermal absorption in the present case, therefore, we are mainly concerned with the translational kinetic energy of molecules. With regard to this process, Milne has suggested a way in which a known mechanism may account for the facts. Klein and Rosseland † have developed the theory of electronic collisions with atoms whose electrons are already in an excited state: the atomic system is restored to its normal state of smaller energy, and the balance, instead of being emitted as radiation, is given by radiationless transfer to increase the kinetic energy of the free electron. Now, given a mass of gas with atoms

* Stewart, *Phys. Rev.*, 1923 Oct. † *Zeit. f. Ph.*, 1921, 46.

already in a state of large energy content due to radiative excitation, a redistribution of this energy by the Klein and Rosseland mechanism to kinetic energy of free electrons would, by the collisions discussed by Stewart, constitute a thermal transformation, without his unknown process which depended on a distrusted mechanism.

This would, however, more correctly be described as an atomic absorption than electronic, as the action of the free electrons is to play a secondary part in the redistribution of the energy after it has been already absorbed by the atomic system. For this reason the absorption corresponding to this thermal transformation cannot add anything at all to the absorption coefficient of the atomic process considered by Milne on the Einstein theory: it can only be considered as providing, within the sphere of activity of the quantum coefficients, a method of redistribution alternative to that seen in the remission of fluorescent radiation. The ratio of these alternatives would be interesting as deciding how far the gas is raised in temperature by the absorption, but it will not increase the amount of energy which becomes absorbed and thus becomes available to either of the two kinds of redistribution, and it is this total amount which is measured by the absorption coefficient.

Though thermal transformation does not thus alter the absorption coefficients which were otherwise calculated, it is important to remember that as an alternative to other modes of redistribution of absorbed energy it will vary with the square and higher powers of the pressure of the gas, as it depends on encounters of two or more bodies; all the other processes we have mentioned depend either on the total gas pressure (scattering and general absorption) or on the partial pressure of the atoms in particular states (monochromatic absorption), and in no case on a higher power. This fact will provide later a useful method of discriminating between the variations of the several processes.

6. It is convenient at this stage to collect the coefficients which have been brought on to a common basis for comparison. The value in each case will be that of the mass coefficient.

i. Total opacity of stellar material to all radiation:

$$K \fallingdotseq 40.$$

ii. Total opacity of homogeneous material to a particular frequency:

$$K_\lambda \fallingdotseq 10^9.$$

iii. Scattering by free electrons in heterogeneous material for all frequencies:

$$s_1 = 1{\cdot}4 \times 10^{-2} N/N_0,$$

where N_0 is the number of molecules per c.c. and N the number of free electrons per c.c.

iv. Scattering by molecules in heterogeneous material for λ 6000:

$$s_2 = {\cdot}6 \times 10^{-4} \text{ (proportional to } 1/\lambda^4).$$

v. Scattering by bound electrons:

$$s_3 = s_2.$$

vi. Quantum absorption by stellar material for all frequencies:

$$\kappa_1 + \kappa_2 \fallingdotseq 5.$$

vii. Quantum absorption by homogeneous material for a particular frequency:

$$\kappa_{1\lambda} \fallingdotseq 10^9.$$

viii. Thermal absorption: a particular type of redistribution of the energy included under quantum absorption.

This summary leaves no room for doubt that in comparing the mass coefficients true absorption, *i.e.* electron transitions within the atomic structure, is predominant in any mass of gas unless it contains such a preponderance of free electrons over atoms as to make quantum absorption a rare event. As before emphasised, in comparing coefficients the homogeneity or heterogeneity of the material and of the radiation is of vital importance, since any electron can scatter, but only atoms in a certain state can absorb; but the concentration factor which might reduce K_λ to K is not large enough to ensure the reduction of κ_λ under the conditions of a heterogeneous atmosphere sufficiently to make s greater than κ, except under special circumstances. This might be considered as confirmed by the evidence of Eddington's absorption coefficient by the Einstein method, of the order of 5 compared with his free-electron scattering coefficient of ·2;* however, appeal to this last is not satisfactory in the present case, as it depended on X-ray frequencies in the interior of a star instead of optical frequencies in the atmosphere.

The problem from which this paper arose has therefore reached this stage of solution: either Schuster's criterion for the appearance of a bright line is invalid so far as it concerns scattering, since s/ρ is always less than κ/ρ, or else there must be some indirect reason why an atmosphere should allow scattering to be more effective even though the mass-scattering coefficient is small.

To investigate this alternative we propose to start from the comparatively known case of the sun, and examine the possible change of effectiveness of the coefficients under different conditions.

Milne has shown† that scattering must be inappreciable in the solar atmosphere compared with absorption, because the two processes have different effects on the law of darkening of the disc towards the limb; scattering darkening would be the same for all wave-lengths, and observation precludes the possibility of this being so in the sun. Lindblad‡ gives other evidence also showing that solar scattering is small. The evidence therefore supports the Schuster theory taking the sun as a normal example in which the predominant absorption results in dark lines. The breakdown in certain other stars of the consequences expected from the data we have compared must be explained, if at all, in the difference between the solar atmosphere and those of bright-line stars.

7. To make clear the physical reasons for this important difference,

* *M.N.*, **84**, 104.

† *Phil. Trans. Roy. Soc.*, 223, A, 1922.

‡ *Ap. J.*, 1923 Sept.

the solar condition will be worked out in this section before turning to the contrasted bright-line stars.

Consider the principal kinds of opacity differentiated by gas pressure, as mentioned in 5. At the comparatively high pressures possible in an outer photosphere, scattering, and absorption with its two alternatives of optical and thermal redistribution, will all be effective in the proportion indicated by their coefficients κ and s. As the pressure falls, passing out beyond the photosphere, all these processes will decrease, but thermal absorption being dependent through collisions on the square of the pressure will fall off more rapidly than either scattering or quantum absorption with radiative re-emission.

Now in the solar chromosphere we have an example of the final state to which this successive elimination of the different processes contributing to the opacity must lead as the gas pressure falls. In the picture of the high chromosphere given by Milne* purely optical absorption is the only mechanism of opacity surviving, and the atmosphere consists of those atoms of calcium which are in a quantum state to be supported by their own absorption of momentum from radiation. Evidence of this was seen by Mitchell at a total eclipse up to 14,000 kilometres above the limb of the sun. Between this upper chromosphere and the last layers to which the concept of temperature as a statistical characteristic of collisions was applicable, there must be some stage at which collisions are so few that the thermal redistribution of absorbed energy is negligible, but scattering is still as active as its small coefficient will allow—the former having fallen behind the latter owing to its dependence on a higher power of the gas pressure. Consider the position this critical stage must occupy: in the sun it is compressed into a narrow zone, as will be seen from the following considerations. Milne † considers that "the absorption lines are formed in the levels where there is an appreciable pressure gradient . . . and where, on the whole, the atoms will be endowed with a Maxwellian distribution of velocities corresponding to a little less than the effective temperature of the star"; *i.e.* the chromosphere is too rarefied to contribute much to visible absorption or visible scattering. Again: "The reversing layer . . . is the region of transition in which equilibrium under an appreciable pressure gradient gives place to low-pressure equilibrium under radiation pressure." ‡ The gradient on the inner or photospheric side, according to Fowler,§ is very steep. These considerations, combined with the fact that the Fowler and Milne use of a mean reversing layer pressure estimate of 10^{-4} atmosphere is verified in spite of the variations to which it might be subject,‖ show that the whole transition layer is in the sun very narrow; and some part of this narrow layer between steep-pressure gradient on the one side and radiative equilibrium on the other side will correspond to the critical stage mentioned, in which scattering has for the moment its maximum comparative effectiveness.

8. Turn now to the physical conditions which in emission-line stars may give a different place to this scattering zone.

* *M.N.*, **84**, 354, 1924. † *Phys. Soc. Proc.*, 1924 Feb., p. 104.
‡ Fowler and Milne, *M.N.*, **83**, 419, 1924.
§ *Phil. Mag.*, 1924 Feb., p. 276. ‖ See Milne, *Phys. Soc.*, *loc. cit.*, p. 109.

Bright lines appear in Q, P, O, B, and Me, and occasionally in other late spectral types. But for the present purpose the following order of classification may be the most convenient:—

(*a*) Certain B stars: those O stars which show narrow bright lines, *i.e.* Harvard Od and Oe classes, included by H. H. Plaskett* in his new O_5–O_9 classification of absorption-line O stars.
(*b*) Wolf-Rayet stars with broad bright bands: Harvard Oa, Ob, Oc, Op, excluded by Plaskett.
(*c*) Apparent O and Wolf-Rayet nuclei in P planetary nebulæ.
(*d*) Me stars and occasionally other rare late-type stars.

The characteristics important for the present problem are as follows:—

i. *Luminosity.*

(*a*) The mean absolute magnitude of measured O stars is $-4{\cdot}0$, with a range of 3 magnitudes.† In all properties Plaskett considers the narrow bright-line O stars as a "continuation of the giant stage of the most massive of the B stars."
(*b*) Plaskett‡ places the mean luminosity of the Wolf-Rayet stars at about 2 magnitudes less than that of the O stars.
(*d*) Wilson§ gives a mean absolute magnitude of $-0{\cdot}9$ for Me, while Merrill‖ mentions the mean brightness at maximum as a hundred times that of the sun.
(*c*) Now, so far bright-line stars all agree in great luminosity; but there is a sudden change in the remaining type: Van Maanen's parallaxes for some planetary nebulæ result in a mean absolute magnitude of $+9{\cdot}1$. This striking faintness of objects whose spectra identify them almost completely with O stars is sufficiently anomalous to be discussed separately below.

ii. *Mass.*

(*a*) J. S. Plaskett¶ gives the mean mass of O stars as of the order of 50 times that of the sun. In the same paper evidence is summarised for a progressive decrease through the B stars whose mean mass is 10 times that of the sun to the A stars of half the mass of B stars.
(*c*) Campbell and Moore** give the masses of some planetary nebulæ as of the order of 50 times that of the sun.
(*b*) Plaskett†† takes it as generally accepted that the above masses refer almost entirely to the nuclei.
(*d*) Merrill‡‡ shows that the mean luminosity and surface brightness of Me stars would mean a mass of 10^6 times that of the sun if the density were the same, or a density of 10^{-6} times that of the sun if the mass were the same. The absence of further data makes it impossible to treat this as more than fairly

* *Publ. Dom. Ast. Obs.*, 1922, No. 30.
† J. S. Plaskett, *Publ. Dom. Ast. Obs.*, 1924, p. 326.
‡ *Loc. cit.* § *Astro. Journ.*, 34, 190.
‖ *Ap. J.*, 1924 March. ¶ *Loc. cit.*, p. 295.
** *Publ. L.O.*, 13, 177. †† *Loc. cit.* ‡‡ *Loc. cit.*

conclusive evidence of low density, together with a presumption in favour of high mass also. It may be remembered that on Eddington's recent theory* the high luminosity is itself physically connected with great mass.

iii. *Density.*—The absolute magnitude of all types except the anomalous P can only be accounted for by low density and large surface even in the case of large masses. Merrill's estimate for Me stars has been quoted, and Plaskett† gives the mean density of O stars as ·01 of the solar density. Since all the stars we are discussing are giants, this O-star density will be larger than in most bright-line stars owing to the stage of condensation reached.

iv. *Temperature.*—On all scales of observational astrophysics, as well as on the Saha and the Fowler and Milne theoretical scales, the bright-line stars we have discussed are all near the hot end of the stellar range except the Me stars, which are the coolest. Two notes must be added to this well-known fact. Firstly, if the Schuster theory is right and the bright-line stars have a scattering atmosphere, it has been shown by Milne‡ that the continuous spectrum will be distorted. This assumes Rayleigh scattering, which is more effective for blue than for red owing to its dependence on $1/\lambda^4$, and will shift the apparent maximum ordinate in the spectral curve towards the red. Spectrophotometric observations of stars nearing the bright-line stage should therefore give too low an apparent effective temperature. Now, Fowler and Milne have temperature calculations for hotter stars§ which do indeed give a higher temperature than that yielded by direct observational methods, though it must be remembered that Wolf-Rayet stars are not proved susceptible to the Saha theory. The independent evidence for a higher temperature for early-type stars than given by observation adds considerably to the probable validity of the scattering theory for bright lines.

The second note on temperature concerns the Me stars, which at first sight seem to contradict the correlation of bright lines with high temperature. Now, if Me stars were spectrally only different from other late stars in having some low-temperature lines bright instead of dark, the class would be more puzzling than it actually is. But two facts beside bright emission make the Me stars stand out from other M stars—namely, long-period variability in all cases, and high excitation of O and P type in a few cases. The latter shows that these cool stars can possess some characteristic equivalent to the source of ionisation in early stars.

9. We are now faced with two problems: firstly, the fulfilment of the bright-line condition in what may be called from the above character-

* *M.N.*, **84**, 308, 1924.
† *Loc. cit.*, p. 328.
‡ *Phil. Trans., loc. cit.*
§ *M.N.*, **84**, 499, 1924.

istics the normal type of emission star, very luminous, very massive, very hot; secondly, the reason for the two outstanding anomalies, the apparent low luminosity of the P class and the apparent low temperature of the Me class. Reserving till a later section the more detailed discussion of these two apparent exceptions, consider what differences between the atmospheric physics of bright-line stars and of the sun should be due to the luminosities, masses, and temperatures.

i. Let g be the acceleration of gravity at the surface of a star. Then

$$g = \frac{GM}{R^2},$$

where G is the gravitational constant, M is the mass, and R is the radius of the star. But the radius is related to the luminosity L and the surface brightness s by the equation

$$L = 4\pi s R^2,$$
$$\therefore \quad g = \frac{4\pi s GM}{L}.$$

Now, taking the sun's absolute magnitude as $+5$ and converting magnitudes into luminosities, the data of the preceding section indicate a far greater increase in L than in M in passing from the sun to the normal bright-line stars even in the case of the most massive. Hence a small surface gravity is a general characteristic of these stars. It must be noticed that as g is a function of M as well as of $1/L$, the minimum value of g will not necessarily be found at the extreme end of the M or L scale.

ii. Next consider the expression $1-\beta$ in the Eddington radiative theory of stellar equilibrium: this depends principally on the mass of the star. Eddington's equation,

$$1-\beta = \cdot 003 m^4 M^2 \beta^4,$$

measures the ratio of the force of radiation pressure to that of gravitation in the equilibrium of the body of the star. If the mean molecular weight m is not changed, Eddington's tables * show the increase in $1-\beta$ in passing from the sun to more massive stars; *e.g.* if M is unity (as in the solar case), $1-\beta$ is ·106, but if M is 13, the value of the ratio is increased to ·615. Hence $1-\beta$ will be large in the cases we have been considering; in fact, many O stars and P nebulæ are well outside the stability limit, giving colour to H. H. Plaskett's suggestion that it is the most massive O stars that develop Wolf-Rayet characteristics, until finally we reach those which are blown out into planetary nebulæ.

iii. Finally, consider that the high-excitation stage seen in all the normal bright-line stars shows that the free electron density is always larger than in the case of the sun.

* *Ap. J.*, 48, 205.

10. Now, though the free electron density is the chief consideration in determining the scattering of radiation, this in itself would not necessarily make scattering predominate over absorption even when allowance is made for the change from monochromatic to general coefficients; but the conjunction in the same star of the other two facts, of low surface gravity in highly luminous bodies and the radiation pressure almost equal to gravity in massive bodies, will result in the wide stretching out in the stars we have considered of that zone which in the outer part of the sun we said was very narrow.

For in the normal type of bright-line star, things will be very different from such as prevail in the sun owing to the different values of g and of $1-\beta$. Remembering that the Eddington equation for $1-\beta$ applies to the body of the star, we may conclude that the close balancing of radiative and gravitational forces which in the sun is only seen in the high chromosphere, will in these stars extend further inwards, and that the last stages of the pressure gradient will be drawn out into a very wide zone instead of falling from an appreciable value to almost zero in the short transition area which in the sun allows absorption lines to develop strongly. The critical stage in which scattering survives beyond thermal absorption will therefore extend over a region wider than in the sun. Where the values of L are very large and the values of M large enough to make $1-\beta$ great without counteracting the effect of L on g, and when the high ionisation increases the scattering on the theory outlined above, it is probable that the Schuster condition may be fulfilled, and provide the explanation in such stars for the appearance of bright lines.

It should be noticed that the physics of the atmospheres of these stars not only allows the Schuster scattering condition more chance of becoming effective, but also gives support in a similar way to the other condition he laid down for the appearance of bright lines, namely, a low-radiation intensity gradient. For the very gradual dying away of pressure characterising these stars will lessen the steepness of the radiation gradient for the following reason: a rapid pressure change will mean a rapid change in the ratio of the two alternatives, thermal and optical, which in 5 were discussed as possible modes of re-emission of absorbed energy. Now if, as in the chromosphere, all the absorbed energy is re-emitted optically instead of by radiationless transfer to kinetic energy, a considerable fraction of the radiation density still goes forward even though at different frequencies, since the fluorescent re-emission is random in direction. If, however, some of the energy is thermally absorbed by the Klein and Rosseland transfer, this is as radiation completely obliterated. The more gradual change from thermal to purely optical transformation of energy will therefore mean that the decrease of total intensity of radiation streams passing outwards will take place more slowly than in the sun. Quantitatively also Schuster's theory here has support. It is known that the Balmer lines, which are often the only ones which appear bright, may appear * as a series of absorption lines increasing in strength towards the violet, with superposed a corresponding series of emission lines decreasing in strength towards the violet. This may show itself in the extreme case of absence

* *Rome Report*, 1922.

of the light component in the violet and absence of the dark component in the red. Now Schuster's consideration of the radiation intensity gradient had predicted : * " With moderate increase of temperature towards the inside of the gas, the shorter wave-lengths are less likely to be bright than the longer wave-lengths."

11. Turn now to those stars whose departure from the series of characteristics given in 8 raise difficulties not so far met in the theory of what we called the normal type of bright-line star of high L, large M, low ρ, and high T.

The present paper has no pretension to attempting a new hypothesis of the remarkably low luminosity (mag. + 9) which the Van Maanen parallaxes give to planetary nebulæ and which seem at first sight to place them outside the Schuster theory. Their great mass and high temperature, and the universal early type of their central stars, make it, however, legitimate and even necessary to suppose that for some reason the light seen by us does not measure the greater emission of energy which must on all considerations be demanded for bodies of such physical characteristics. Jeans † shows that some abnormal cutting down of light is indicated by the fact that brightness is at a maximum at the outside edge of many planetary nebulæ instead of falling gradually to zero, and that we appear in some cases only to have emission from the inner and outer surfaces of a shell. Jeans also points out that since the wave-length of maximum intensity is in the ultra-violet for planetary nebulæ, the main radiations do not reach us. The demand for some explanation of this type shows that the faintness here is not a normal faintness which could legitimately be contrasted with the brightness of the normal emission-line stars, and that it may be assumed that there need be no conflict from this consideration with the theory we have outlined.

In the other departure from the normal type of bright-line star, the Me stars only fail to complete the combination of required physical properties by their low temperature, and it must be remembered that temperature was the least important of the factors mentioned at the beginning of 10. Now the function of temperature in the theory was as an ionising agent to increase the free-electron density and so add to the scattering. But although temperature is the most familiar stellar, ionising agency, Fowler has shown ‡ that even in thermodynamic equilibrium radiative ionisation is often more effective at low pressures. In Me stars the occurrence occasionally of He^+ lines (in T Coronæ) and nebular lines (in R Aquarii) shows that though the ionisation proper to the TiO_2 association temperature is low, there must nevertheless be also an ionisation beyond the temperature of this medium not accounted for by thermodynamic equilibrium. This ionisation may be radiative and not necessarily a matter of collisions or temperature. The suggestion that these different stages of ionisation—the one discordant with the theory of the normal emission-line star, the other agreeing with it—demand the separation of two areas in the star, is borne out by Slipher's discovery of a nebulous envelope to R Aquarii.

Apart from these rare cases of high excitation, the bright-line Me stars only differ from the dark-line M stars in being long-period variables

* *Loc. cit.*, p. 16. † *M.N.*, 83, 9. ‡ *Phil. Mag.*, 1924 Feb.

and in all being giants.* Now the normal bright-line star theory makes use of the giant characteristic, and the sole remaining difference—the absence of apparent high excitation for all except those few which show He^+ or nebular lines—must therefore be explainable, if at all, by whatever mechanism is assumed for the variability. Consider the two main current hypotheses for this. The "Geyser" theory † of the periodic bursting of a viscous cover which is reformed at minimum seems the most satisfactory hypothesis of variability, as it gives some reason for the most striking characteristic of these stars, namely, the relative approach velocity of the material of the bright lines at maximum. ‡ This would allow, on the present thesis, for the fulfilment of the Schuster conditions as soon as the violently projected gas had spread itself out far enough to reproduce temporarily at maximum the state which in the normal type of emission star persists as a permanent and stable characteristic. The "Veil" theory § of a calcium screen condensing at minimum would not so readily bring these stars into line with the normal emission type. But this theory is also unable to explain the crucial fact of the approach velocities; its chief merit is that absorption by a calcium cloud would explain the faintness of the Balmer H_ϵ line by the proximity of the calcium H line. Now the properties of the calcium atom have been shown both by J. S. Plaskett || and Milne ¶ to tend to the existence and even the visibility of such clouds entirely apart from any question of variable stars, and it may well be that the "Veil" theory is true in the case of Me stars among many others. But if the Me stars have not a monopoly of calcium clouds, we fall back on the more hopeful "Geyser" theory for the variability, which also allows these stars a loophole for inclusion in the theory of normal emission-line bodies.

12. *Summary of the Paper.*—The recent sources of evidence as to scattering and absorption mechanisms are critically examined to find the limit within which may vary the values of different kinds of opacity coefficient for application to Schuster's theory of emission-line spectra. It is found that with the resulting mass coefficients the ratio s/κ can never reach the value needed by Schuster's criterion. The validity of this emission-line theory is therefore sought instead in some gravitational and radiative peculiarities of the atmospheres of these stars; and a condition of transition between one kind of opacity and another is suggested, which in the sun is confined to a narrow zone, but in bright-line stars is wide enough to allow a large effective scattering even with a small coefficient. The reasons for the existence of this transition zone are worked out from recent data of mass, luminosity, density, temperature, and the possible place in the theory of the anomalous P and Me spectra is discussed.

I am grateful to Mr. F. J. M. Stratton for the kind interest he has shown in this paper, and for his advice and help.

Physics Department, Birmingham University:
1924 *September.*

* See *Rome Report*, 1922; also Adams, *Nat. Acad. Proc.*, 1916, **2**, 157.
† See Campbell, *Stellar Motions*, p. 303.
‡ See Merrill, *Ap. J.*, 53, 185.
§ Merrill, *Publ. Ast. Obs. Mich.*, 1916, **2**, 70.
|| *M.N.*, **84**, 2.
¶ *M.N.*, **84**, 354.

On Some Relations between the Solar Constant and Solar Activity.

By Issei Yamamoto.

Abstract.—Very recently C. G. Abbot found some relations between the fluctuations of the solar constant and the spot apparitions on the solar disc. I examined the rich materials from Mount Wilson spectroheliograms to compare them with daily records of the solar-constant observations, and came to some conclusions which fairly confirm Abbot's statements. For complete explanations of the phenomena it is evident that we should consider the "age" of a sun-spot in its development, the effect upon the solar constant depending on the rate of growth as well as its position on the solar disc. These effects are, however, entirely individual, and do not yield any statistical results. Some quantitative investigations show that there is practically no relation between the solar constant and the area of calcium flocculi on the sun, although a sudden diminution of the solar constant was evident during the sun-spot minimum of the last year or two.

1. *Introduction.*—Last summer, while at the Mount Wilson Observatory, I made a short investigation of a possible relation between the solar constant and other manifestations of solar activity.* Dr. C. G. Abbot supplied me with the final reduced values of the solar constant at the stations of both hemispheres, and Dr. W. S. Adams put at my disposal the results of solar observations made at the Mount Wilson Observatory. The latter material consisted of spectroheliograms and observations of the magnetic polarity of sun-spots.

Before I started my investigations, Dr. Abbot gave me a short note containing a summary of his comparison of solar phenomena, in which he noted the following "four points":—†

1. New or increased spots connected with higher values of the solar constant;
2. Central spots resulting in lower values of the solar constant on the next day;
3. Great activity of the solar surface connected with hydrogen H_α, connected with high values of the solar heat; and
4. Long quiet period of the solar surface connected with falling values of the solar constant.

2. *Comparison of the Solar Constant and the Activity on the Sun's Surface.*—It is fairly well known that the calcium flocculi precede and outlive the sun-spots which are associated with them, so that spots may be considered as the *peak* phenomena of the flocculi on the solar surface. Accordingly I made comparisons of the daily solar-constant values and the calcium flocculi on the sun, to check Dr. Abbot's "four points" with flocculi instead of spots. The results of these examinations were not conclusive, showing some contradictions as well as some confirmations

* More details will be published in another paper by me in Japan.

† These have been published by Dr. Abbot later in *Proc. Nat. Acad. Science*, 9 (1923 October).

of the "four points" above mentioned, when calcium flocculi are concerned.

Next I turned to the magnetic observations of sun-spots for other material to be compared with the solar constant. This time the results were fairly definite in showing some correlation. I found that sometimes sun-spots apparently increased the solar radiation, and at other times decreased it, the effect being more conspicuous when the spots came very near to the apparent centre of the solar disc. A sun-spot which is growing in magnetic activity seems to cause the solar constant to increase, while a sun-spot whose activity is decreasing is accompanied by a decrease in the solar constant. Of course these phenomena were shown best in special cases, which were limited in number. There were many cases in which two or more groups of sun-spots were on the sun at the same time, and their respective effects interfered with one another, so that the variation of the solar constant was too complicated to be explained easily. Moreover, even a single group of sun-spots was sometimes so complex that its effect upon the solar constant was not apparent. Generally the effect of a spot-group near its maximum magnetic development was not simple, and counter-actions of increase and decrease of heat seem to have been taking place, so that the curve of the solar constant at that stage did not show any decided characteristic, indicating apparently a sort of compensation of plus and minus actions. Consequently, I should say that it is generally more difficult to decipher the solar-constant curve in terms of sun-spot activity during the epochs of maximum than during those of minimum activity.

It seems to me that the "age" of a sun-spot should be considered in such investigations. Since I was using the Mount Wilson magnetic observations in this connection, it was apparent that the magnetic fields were strongest in the large spots and that the area and field strength were closely related. In fact, it may be that the strength of the magnetic field or some other related quantity is more fundamental than the area. Certain types of geometrical figures of sun-spots, and even of calcium and hydrogen flocculi, might be used with more or less significance for such problems.

3. *Statistical Investigations of Correlation between the Solar Constant and Calcium Flocculi.* — So far the comparisons of the solar constant with other phenomena were made particularly with respect to their short-lived variations. The next step was to investigate the relations between the secular or long-period variations of the phenomena and the solar constant. For this purpose every spectroheliogram of calcium flocculi since July 1918 was examined, and the apparent area of the flocculi estimated on an arbitrary scale of 0 to 10. The results are given in the following table:—

TABLE I.

Months.	1918.		1919.		1920.		1921.		1922.		1923.	
	S.	f.	S.	f.	S.	f.	S.	f.	S.	f.	S.	f.
Jan.	...	...	1·944	4·8	1·964	3·8	1·962	3·0	1·944	2·2	1·911	1·1
Feb.	...	...	1·948	5·2	1·956	4·4	1·950	2·6	1·947	2·2	1·903	1·1
Mar.	...	...	1·941	5·0	1·946	4·0	1·943	2·9	1·931	2·7	1·908	1·0
Apr.	...	...	1·952	4·5	1·952	2·9	1·944	2·8	1·923	2·1	1·923	1·4
May	...	...	1·940	4·8	1·953	3·3	1·948	2·5	1·926	1·8	1·919	0·7
June	...	...	1·955	5·2	1·939	3·5	1·937	2·5	1·918	1·4	...	1·2
July	...	...	1·954	5·1	1·945	2·8	1·943	2·8	1·911	1·7	...	...
Aug.	1·954	4·8	1·953	5·1	1·929	1·9	1·937	2·1	1·919	1·1	...	...
Sept.	1·944	4·5	1·939	4·9	1·947	2·0	1·944	2·1	1·914	1·0	...	...
Oct.	1·935	5·4	1·952	4·1	1·945	3·7	1·947	2·2	1·922	1·6	...	...
Nov.	1·942	5·0	1·953	4·0	1·950	3·7	1·954	2·0	1·915	1·2	...	...
Dec.	1·959	5·3	1·952	4·6	1·952	3·6	1·950	1·8	1·920	1·3	...	...

In this table "S" is the monthly mean value of the solar constant observed at the stations of the Smithsonian Institution and "f" is the monthly mean value of the relative area of the calcium flocculi estimated

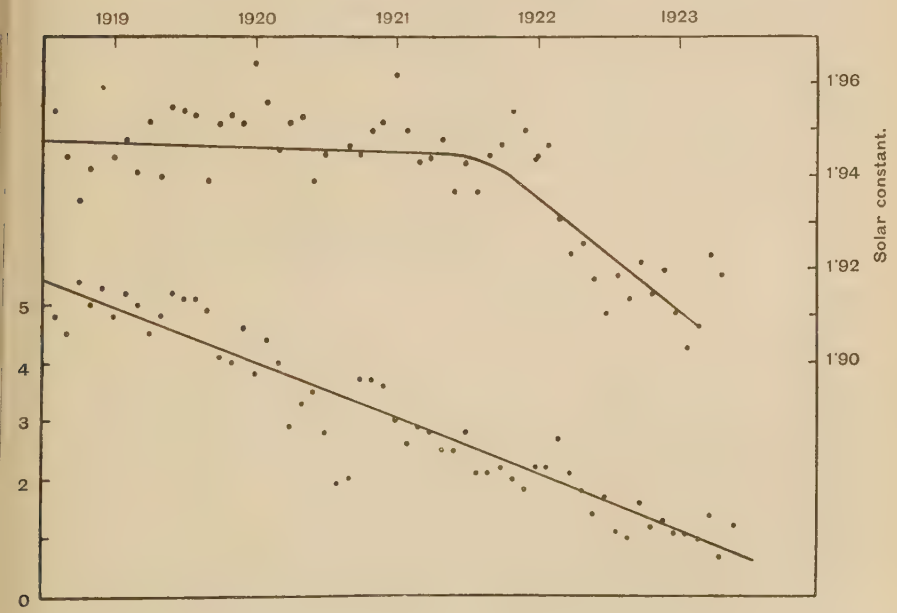


FIG. 1.

from the Mount Wilson spectroheliograms. Fig. 1 shows the same data diagrammatically. A more or less regular fluctuation with an annual period is seen in the solar constant. For my purpose, a series of "corrections" to compensate this annual variation was deduced thus:

TABLE II.

Corrections for the Annual Variation of the Solar Constant.

Month.	Corrections.	Smoothed Values from Three Consecutive Values.	Month.	Corrections.	Smoothed Values from Three Consecutive Values.
January	− ·009	− ·007	July	+ ·001	+ ·004
February	− ·003	− ·002	August	+ ·005	+ ·005
March	+ ·005	+ ·001	September	+ ·005	+ ·005
April	+ ·001	+ ·002	October	+ ·003	+ ·002
May	+ ·001	+ ·002	November	− ·002	− ·001
June	+ ·004	+ ·002	December	− ·005	− ·006

From a theoretical consideration the quantities of the last column were increased by 20 per cent., because they have suffered a diminution of amplitude by their peculiar treatment. These annual "corrections" were applied to the values of the solar constants in Table I., which was subsequently condensed into Table III.

TABLE III.

Mean Normal Solar Constant and Calcium Area for each Quarter Year.

Months.	1918.		1919.		1920.		1921.		1922.		1923.	
	S.	f.	S.	f.	S.	f.	S.	f.	S.	f.	S.	f.
Jan.–Mar.	...	...	1·942	5·0	1·953	4·1	1·949	2·8	1·938	2·4	1·905	1·1
Apr.–June	...	...	1·951	4·8	1·950	3·2	1·945	2·6	1·924	1·8	1·923	1·1
July–Sept.	1·954	4·6	1·953	5·0	1·943	2·2	1·940	2·3	1·919	1·3	...	...
Oct.–Dec.	1·944	5·2	1·951	4·2	1·948	3·7	1·949	2·0	1·918	1·4	...	...

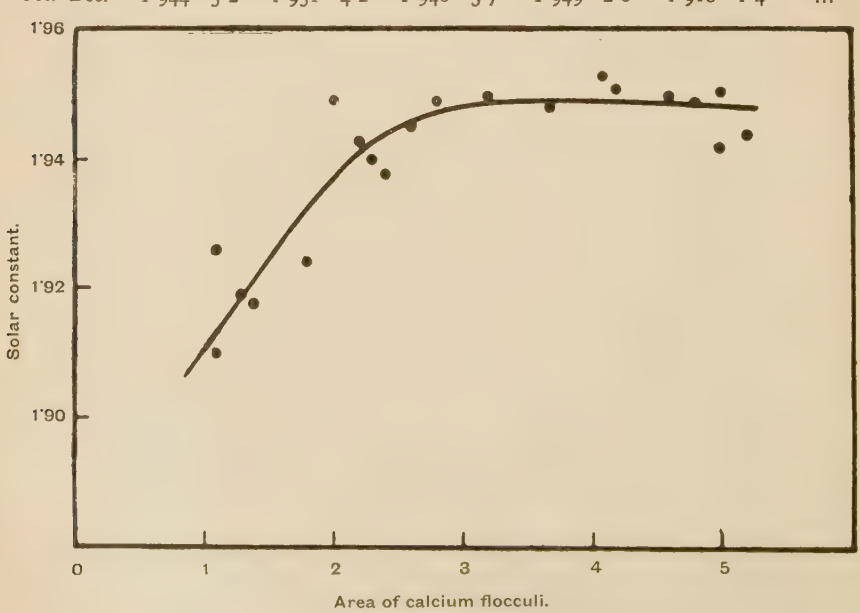


FIG. 2.

This table is shown graphically by fig. 2, in which the flocculi and the solar constants are represented as abscissæ and ordinates respectively. There appears to be a correlation, but a close examination shows that while the decrease in the calcium areas was uniform over this interval, the

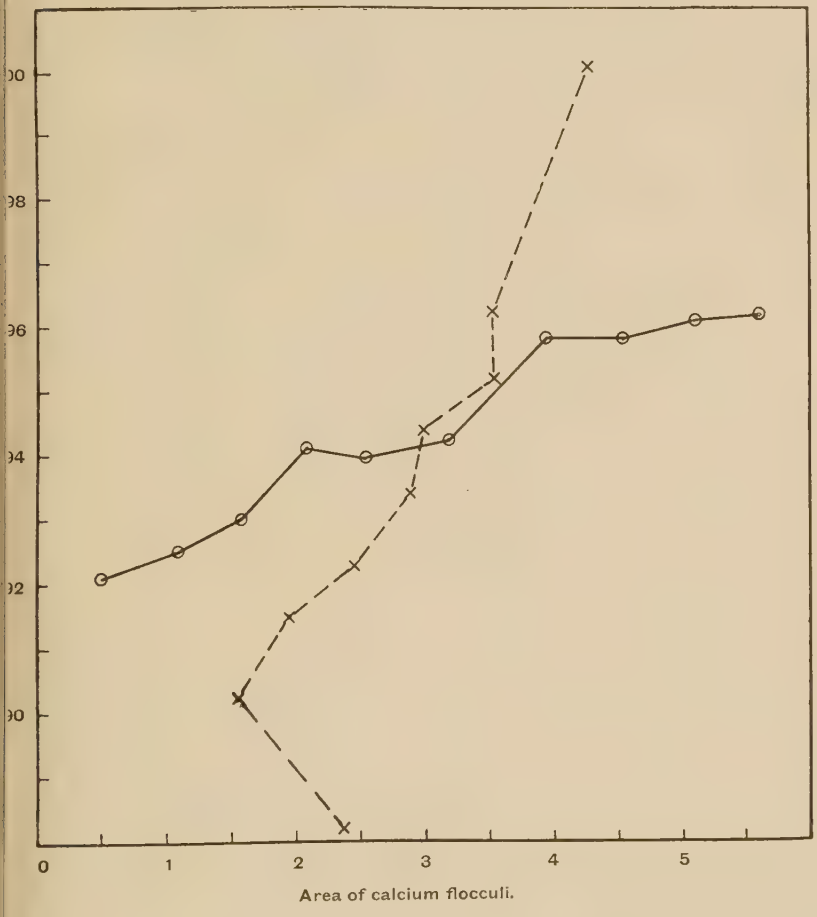


Fig. 3.

drop in the solar constant was abrupt, so that one cannot get a definite conclusion about the numerical relation between the two phenomena.

Correlation curves were made by getting the mean value of the solar constant within certain limits of the estimated areas of the calcium flocculi, and also the mean value of the flocculi areas for certain intervals of the solar constant. Fig. 3 shows the resulting curves, the circles representing the mean solar constants for consecutive intervals of the flocculi area, and the crosses the similar values of the flocculi for

consecutive intervals of the solar constant. I next proceeded to a more detailed analysis by this process, that is, to make similar computations *for each year*. The final results are in the following two tables.

TABLE IV.

Year	1·0	1·0 − 1·4	1·5 − 1·9	2·0 − 2·4	2·5 − 2·9	3·0 − 3·4	3·5 − 3·9	4·0 − 4·4	4·5 − 4·9	5·0 − 5·4	5·5
1918	...	...	...	...	...		1·935		1·949	1·939	1·945
	...	...	...	..	...		3·21		4·63	5·07	5·66
	...	...	...	...	...		12		12	30	29
1919	...	...	...		1·953			1·953	1·952	1·950	1·954
	...	...	...		2·77			4·04	4·60	5·08	5·74
	...	...	...		9			16	27	72	26
1920	...	1·936	1·944	1·947	1·946	1·949	1·952	1·948	1·950		1·943
	...	1·13	1·61	2·16	2·63	3·13	3·55	4·12	4·60		5·42
	...	18	17	26	24	31	21	21	24		21
1921		1·948	1·951	1·951	1·941	1·943	1·944	1·945		...	...
		0·96	1·58	2·11	2·61	3·10	3·59	4·08		...	...
		28	19	46	30	49	18	10		...	...
1922	1·922	1·920	1·915	1·925	1·932		1·925			...	...
	0·55	1·13	1·59	2·13	2·55		3·37			...	...
	36	51	29	40	20		18			...	...
1923	1·918	1·909		1·915			...	...	...	...	...
	0·45	1·13		1·72			...	...	...	...	...
	28	27		11			...	..	...	...	...

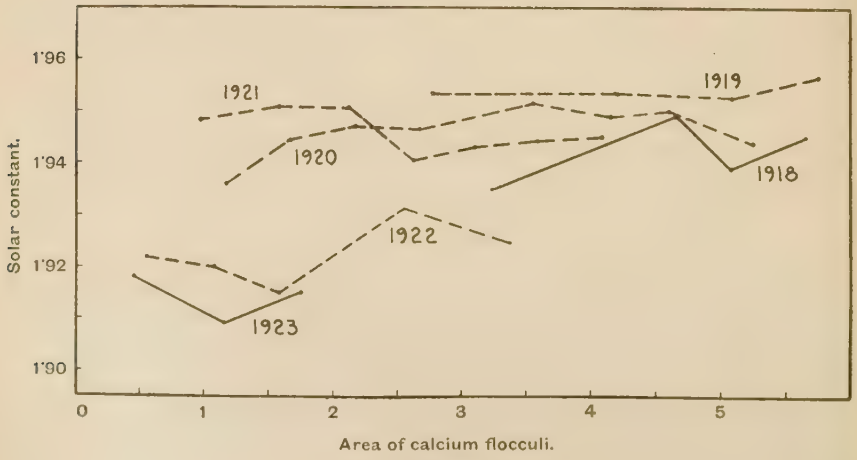


FIG. 4.

TABLE V.

ear	1·899	1·900 – 1·909	1·910 – 1·919	1·920 – 1·929	1·930 – 1·939	1·940 – 1·949	1·950 – 1·959	1·960 – 1·969	1·970
)18		1·895		1·925	1·934	1·945	1·953	1·964	1·982
		5·18		5·14	4·85	4·92	4·86	5·04	5·29
		12		10	10	15	19	7	10
)19		1·923			1·935	1·945	1·955	1·963	1·977
		3·96			5·05	4·85	4·77	4·82	5·04
		17			17	24	41	37	14
)20		1·904		1·924	1·935	1·945	1·953	1·964	1·984
		3·04		2·70	3·08	3·07	3·34	3·28	3·65
		11		19	30	51	50	31	11
)21		1·904		1·924	1·935	1·944	1·954	1·963	1·982
		2·92		2·89	2·37	2·48	2·22	2·40	2·43
		10		13	32	65	41	30	9
)22	1·880	1·905	1·915	1·922	1·934	1·944	1·957		...
	1·57	1·40	1·67	1·51	1·81	1·94	1·74		...
	15	21	40	46	40	24	S		...
)23	1·884	1·905	1·914	1·925	1·937		...	...	...
	1·09	0·88	1·08	0·82	0·89		...	...	...
	8	18	14	17	9		...	...	...

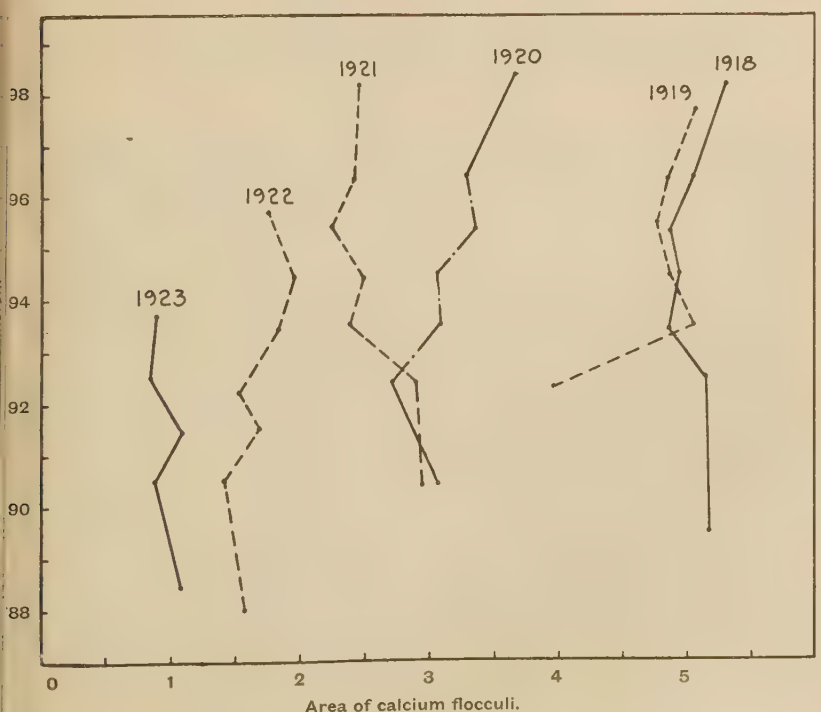


FIG. 5.

In these two tables, the first line of each row shows the mean values of the solar constant, the second line those of the area of flocculi, and the third line the number of observations used in obtaining the means. Cases in which both flocculi and solar constant were not observed on the same date are *not* taken into account.

Figs. 4 and 5 show the results of the above Tables IV. and V. Now we see that there is no correlation between the solar constant and the calcium flocculi within each year, for the curves for each year are practically parallel to one another and to the co-ordinate axes, or, in other words, the mean values for each argument are sensibly constant. There is, however, a general tendency for low values of the solar constant to occur more frequently in later years, as is shown in Table III., which circumstance is also responsible for the slope of the curves in fig. 3.

The question why this sudden drop in the solar constant should occur as the time of minimum calcium activity (and of minimum spot activity) approaches, when at other phases there seems to be no connection between the calcium arcs and solar constant, requires further study. Here, as in the case of sun-spots cited in the first part of this paper, it may be that at maximum activity counteracting effects hide the relations which during the epoch of minimum are more conspicuous.

1924 *June* 30.

Note on Expressions for the Measurement of the Effect of Diffraction and the Presence of Optical Aberrations on the Images of Point, Line, and Plane Luminous Sources. By Arnold Buxton, M.A.

The problem of determining the intensities of light in the diffraction patterns of a point-source of light, such as a star, has been previously considered in the ideal case (*i.e.* the Airy spurious disc) and in the presence of malfocus, spherical aberration, coma, and astigmatism. The method adopted was to consider each aberration present in turn to the exclusion of all the others, and investigate its effect on the character of the point-source image. Then a few combinations of aberrations were considered, but chiefly malfocus and spherical aberration, from which was deduced the fact that the best image lies in a plane perpendicular to the optical axis, midway between the marginal and paraxial foci. In other words, if there is a residual of spherical aberration in an optical system, we can neutralise it by an equal amount of malfocus, so that we obtain a disc image practically identical with that of the ideal Airy spurious disc, where only the diffraction defect is considered.

So, in the first part of this note, the method has been similarly to consider the diffraction and chromatic aberration defects, assuming that all the other aberrations are absent.

Neglecting chromatic differences of magnification, the coloured foci will be distributed along the optical axis, and with reference to one coloured pencil all the others may be considered as being affected with malfocus. As it is agreed that the yellow focus is the best for visual

purposes, we shall first consider the disc-image in a plane perpendicular to the optical axis through the yellow focal point. The other coloured rays will then suffer from malfocus. The intensities for malfocus conditions of a point-source have been investigated in a previous paper.* The formulæ are

$$C = \int_0^1 J_0\left(\frac{ry_1}{f}\right)\cos cr^2 \,.\, \pi dr^2$$

and

$$S = \int_0^1 J_0\left(\frac{ry_1}{f}\right)\sin cr^2 \,.\, \pi dr^2.$$

With (Intensity) $= C^2 + S^2$, where J_0 is the Bessel Function of zero order, r the radius of the aperture, f the focal length, y_1 is the distance of the point where the intensity is considered from the centre of the image, and c varies directly as the amount of malfocus or as the distance between the two foci. In the case of monochromatic light, the quantity c only depends on the position of the focal plane, and will only vary according as we go in or out of focus. For any one position it will be constant, but in the case of coloured rays c will change for every colour. In other words, c will be a function of the wave-length $c = f(\lambda)$. Hence the intensity formulæ will become

$$C = \int_A^H \int_0^1 J_0\left(\frac{ry_1}{f}\right)\cos f(\lambda) r^2 \,.\, \pi dr^2 d\lambda,$$

$$S = \int_A^H \int_0^1 J_0\left(\frac{ry_1}{f}\right)\sin f(\lambda) r^2 \,.\, \pi dr^2 d\lambda,$$

with $I = C^2 + S^2$, where A to H denotes the range of wave-lengths considered. The function $f(\lambda)$ of the wave-length must now be further investigated. The refractive indices for the different colours are given by an amendment † of Cauchy's formula

$$n_\lambda = n_0 + \frac{\nu_1}{\lambda^2} + \frac{\nu_2}{\lambda^4} + \ldots$$

as

$$n_\lambda = n_0 + \frac{\nu_1}{\lambda} + \frac{\nu_2}{\lambda^{3\frac{1}{2}}},$$

where n_0, ν_1, ν_2 are constants to be determined for any particular glass but which will not cover more than one type of glass. These constants are, however, more conveniently expressed from the spectrometer data supplied by the maker. Chance's list, for instance, gives for Crowns and Flints

$$N_d, \quad N_f - N_c, \quad \frac{N_d - 1}{N_f - N_c}, \quad N_f - N_d, \quad N_g - N_f.$$

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* *Monthly Notices R.A.S.*, 1921 June.

† Due to Prof. A. E. Conrady.

Hence n_λ can be written as

$$n_d + a\left(\frac{1}{\lambda} - \frac{1}{\lambda_d}\right) + b\left(\frac{1}{\lambda^{\frac{1}{2}}} - \frac{1}{\lambda_d^{\frac{1}{2}}}\right),$$

where n_d, λ_d denotes refractive index and wave-length for the D-line and a and b are constants.

$$a = 4{\cdot}9101(N_f - N_c) - 6{\cdot}5728(N_{g_1} - N_f),$$
$$b = -{\cdot}199435(N_f - N_c) + {\cdot}43181(N_{g_1} - N_f).$$

Now $c = \Sigma(d - D)dn$ [Σnd is the length of the paraxial path, ΣnD that of the marginal, and $\Sigma(d - D)dn$ the difference between consecutive coloured paths], or if the lens system be considered as made up of thin lenses, $c = \Sigma\frac{F}{\nu}$ where F is the power and ν the dispersion factor $\frac{N_d - 1}{N_f - N_c}$.

As $f(\lambda)$ can only be obtained empirically, and as the intensity integrals are unwieldy, they will no doubt be most readily evaluated by the methods of approximate integration for double integrals (either by a double application of one of the Simpson rules or by Burnside or Sheppard's formulæ). A rougher approximation can be obtained when the effect of the secondary spectrum is considered, for the dispersion at the G end of the spectrum will be zero by the choice of the wave-length for the brightest focus. In this case we need only consider the dn for the F and C lines ($dn = N_f - N_c$).

The integrals may be used to determine the resolution in any focal plane when there is no correction of the chromatic aberration, and the other aberrations are assumed to be absent. In the case of the yellow focal plane, the coloured rays will distribute themselves round the focus in "aberration discs," as Helmholtz called them, when focussed on the retina of the eye.* The intensities in these discs can be numerically determined, and different values obtained for other focal planes. A more profitable investigation, however, would be to calculate the effect of the secondary spectrum in the focal plane for the brightest focus. The yellow and dark green, the orange and the blue, and the red and indigo would unite at increasing distances from the brightest focus, according to the magnitude of the secondary spectrum. The photographic and astrographic problem could then be considered by the displacement of the C and F union to that for the F and G′.

Mr. B. Mourashkinsky of Petrograd has communicated to the *Phil. Mag.*, 46, 1923 Nov., a paper which collects and extends the work of Lommel and Struve (outlined in Gray and Mathews' treatise on the Bessel Functions) from the luminous point-source to the luminous line-and-plane sources. Such an investigation is of paramount importance in consolidating the theory of optical resolution. While the results give a good guide as to the limits of resolution, the inquiry is based on the same assumptions that Sir G. Airy used in 1834, and no account is taken of the effects of the different aberrations. For the luminous

* This aspect of the effect of chromatic aberration only has been treated in a paper by Dr. H. Hartridge to the *Journal of Physiology*.

point-source, such as a star (and for central rays), our investigations up to date seem to show that the light intensity in the disc-image can be written in the form

$$I = C^2 + S^2$$

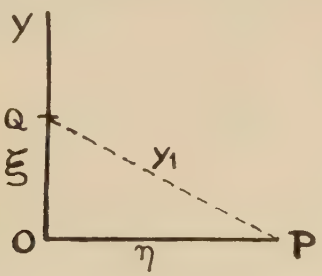
where

$$C = \int_A^H \int_{r=0}^{r=1} J_0\left(\frac{ry_1}{f}\right) \cos f_1(c_1 r^4 + cr^2) \,.\, \pi dr^2 d\lambda$$

and

$$S = \int_A^H \int_0^1 J_0\left(\frac{ry_1}{f}\right) \sin f_1(c_1 r^4 + cr^2) \,.\, \pi dr^2 d\lambda.$$

f_1 represents some function of λ and denotes the chromatic aberration effect, c_1 and c are constants independent of λ and r denoting the lengths of the spherical and malfocus aberrations, and A to H denotes the range of wave-lengths.



These integrals can now be extended to include the intensities in the lamina images due to the luminous line-source by the use of triple integrals. Further, an expression can then be obtained for the luminous plane-source, thus completing the problem for the central aberrations.

Consider the intensity at a point P distant η from the perfect reproduction OY of the luminous line-source. Then, owing to the diffraction effect of an element of the line reproduced at Q, the intensity at P, assuming malfocus, spherical and chromatic aberration present, will be given by $I = (C^2 + S^2)(d\xi)^2$, where C and S have the values already assigned with $PQ = y_1$. Calling $OQ = \xi$, $y_1^2 = \eta^2 + \xi^2$. Similarly for any other element along OY. The sum of all these effects due to successive elements gives the intensity at P due to the luminous line as $C_1^2 + S_1^2$, where

$$C_1 = \int_{\xi=-\alpha}^{\xi=+\alpha} \int_A^H \int_{r=0}^{r=1} J_0\left(\frac{r}{f}\sqrt{\eta^2+\xi^2}\right) \cos f_1(c_1 r^4 + cr^2) \,.\, \pi dr^2 d\xi d\lambda,$$

and S_1 is the same expression with sine instead of cosine. The limits of resolution of two luminous lines would then correspond to the value of η corresponding to the first zero of $C^2 + S^2$. (α is a constant depending on the length of the luminous line.)

For luminous plane-sources with OY as the dividing line, the intensity at P is given by $C_2^2 + S_2^2$, where

$$C_2 = \int_0^{\eta_1} C_1 d\eta \quad \text{and} \quad S_2 = \int_0^{\eta_1} S_1 d\eta,$$

η_1 denoting the limit of the value of η measured in the opposite direction to the η in the figure. Here, again, the limits of the separation of two luminous planes of the same brightness, bounded by OY and a parallel line, can be determined from the first zero of $C_2^2 + S_2^2$. Previous investigations seem to show that, if the resolution in the non-central part of the field has to be determined and coma is present, the above formulæ apply with $J_0\left(\frac{r}{f} y_1\right)$ replaced by $J_0\left(\frac{r}{f} y_1 + \mu r^3\right)$, where μ depends on the length of the coma figure. The presence of astigmatism complicates the problem still further, although its separate effect might be investigated by extending the formulæ given in the *Monthly Notices of R.A.S.*, 1923 June, to luminous line and plane-sources.

The Technical College,
Cardiff.

Stellar Parallaxes determined at the Royal Observatory, Greenwich.

(*Communicated by the Astronomer Royal.*)

The following list is a continuation of the results obtained with the 26-inch telescope and published annually in the November number of the *Monthly Notices* since 1920.

In the column Proper Motion the first value given is from meridian observations; the second is the relative proper motion derived from the photographs. The motions in both R.A. and Dec. are expressed in arc of a great circle.

In calculating the parallax, the relative proper motion has been used, *not* the meridian proper motion.

The Probable Error has been calculated by the formula

$$r = 0{\cdot}6745 \times \sqrt{\frac{[vv]}{m - \mu}} \times \frac{1}{\sqrt{w}}.$$

The first given is the P.E. of the parallax, the second is the P.E. of the relative proper motion and applies to both R.A. and Dec. when the latter is given.

Generally six comparison stars of mean magnitude 11–12 have been used, the magnitude of the parallax star being reduced to equality by means of the rotating sector.

For the brighter stars a neutral-tinted filter has been employed in onjunction with the sector.

No.	Name and B.D. No.	Mag. Vis. Photog.	Spectral Type.	R.A. Dec. 1900·0.	Proper Motion. R.A.	Proper Motion. Dec.	Parallax.	P.E.	No. of Plates.
		m		h m s	″	″	″	″	
1	O.A. 2034	8·8	Go	1 44 41	+·304	−·070	+·013	±·007	25
	65° 209	9·2		+65° 57′	+·261	−·084		±·002	
2	Hels. 1636	9·4		1 45 5	+·070	−·020	+·012	±·007	22
		10·6		+64° 13′	+·059	−·022		±·002	
3	O.A. 2618	8·5		2 14 2	+·493	−·170	+·040	±·006	27
	70° 169	9·4		+70° 43′	+·537	−·168		±·002	
4	79° 81	9·4		2 34 12	+·189	−·122	−·004	±·008	20
		10·7		+79 50′	+·182			±·007	
5	73° 168	5·2	A2	3 1 5	+·029	−·083	+·028	±·012	20
		4·8		+74° 1′	+·027			±·011	
6	68° 224	9·4		3 2 44	+·202	−·115	+·001	±·006	22
		9·8		+68° 52′	+·175			±·005	
7	78° 114	9·5		3 14 25	+·191	−·087	+·014	±·005	22
		10·8		+78° 22′	+·189			±·005	
8	79° 126	8·3	F8	3 52 26	+·192	−·036	+·010	±·006	21
		8·5		+79° 20′	+·200			±·005	
9	68° 302	9·5		3 53 9	+·032	−·210	−·010	±·009	16
		9·9		+68° 31′	+·061			±·009	
10	66° 314	9·2		4 5 56	+·209	−·105	+·009	±·007	22
		9·5		+66° 11′	+·179			±·007	
11	72° 245	9·5		4 44 32	−·156	+·244	+·015	±·006	25
		10·4		+72° 47′	−·141			±·005	
12	O.A. 5529	7·0	Ko	5 3 19	+·030	−·104	−·003	±·006	21
	69° 307	8·4		+69° 42′	+·016			±·008	
13	81° 183	8·6	Go	5 14 25	+·097	−·145	−·002	± 005	18
		9·1		+81° 37′	+·105			±·007	
14	84° 112	9·0	G	5 43 38	+·059	+·166	+·009	±·005	24
		8·9		+84° 59′	+·081			±·007	
15	67° 420	9·0	Ko	6 8 6	−·123	−·004	+·022	±·005	28
		8·7		+67° 46′	−·153			±·005	
16	64° 654	8·0	Ko	7 42 4	−·085	−·076	+·010	±·005	18
		8·1		+64° 21′	−·077			±·006	

No.	Name and B.D. No.	Mag. Vis. Photog.	Spectral Type.	R.A. Dec. 1900·0.	Proper Motion. R.A.	Proper Motion. Dec.	Parallax.	P.E.	No. of Plates.
		m		h m s	″	″	″	″	
17	80° 245	9·5		7 54 58	+·147	−·421	−·003	±·009	23
		10·0		+80° 12′	+·135			±·011	
18	73° 447	9·5		9 0 9	−·143	−·271	+·046	±·006	26
		10·9		+73° 50′	−·154			±·005	
19	77° 361	9·2		9 6 16	−1·060	−·028	+·038	±·005	20
		10·9		+77° 40′	−1·063			±·003	
20	L.F. 1466	7·5	G5	9 12 0	− ·126	−·293	+·026	±·005	27
	65° 703	8·1		+65° 26′	− ·134	−·291		±·003	
21	77° 374	9·5		9 25 5	+ ·092	−·220	+·029	±·005	18
		11·0		+77° 1′	+ ·104			±·005	
22	65° 737	9·2		9 42 0	− ·389	−·186	+·018	±·005	23
		9·8		+65° 47′	− ·420			±·003	
23	75° 403	9·3		10 1 42	+ ·244	+·299	+·051	±·008	21
		10·7 10·6 }		+75° 37′	+ ·239			±·005	
24	71° 632	9·5		12 46 11	− ·185	−·185	+·036	±·005	20
		10·5		+71° 44′	− ·228			±·005	
25	67° 775	9·5	K	13 6 7	− ·146	+·065	+·030	±·007	22
		10·9		+67° 1′	− ·132			±·007	
26	Star near 66° 4140			13 49 58	− ·566	−·131	+·043	±·008	23
		13·0		+66° 2′	− ·565			±·006	
27	65° 995	8·8	Ko	14 22 38	− ·146	+·090	+·004	±·007	22
		9·4		+65° 36′	− ·150			±·006	
28	85° 254	9·4		15 18. 51	− ·016	−·204	−·007	±·007	25
		10·2		+84° 55′	− ·001			±·005	
29	66° 916	8·9	K	15 39 38	+ ·064	−·191	+·015	±·007	26
		9·6		+66° 10′	+ ·051			±·006	
30	64° 1102	9·1		15 55 9	− ·188	+·189	+·041	±·005	25
		10·3		+64° 6′	− ·114			±·004	
31	78° 565	8·0	Ko	16 43 53	− ·103	+·100	+·008	±·011	21
		9·0		+78° 3′	− ·068			±·005	
32	69° 900	8·1	Ko	17 13 10	+ ·013	−·150	+·023	±·012	30
		8·7		+69° 25′	+ ·008			±·008	

	Name and B.D. No.	Mag. Vis. Photog.	Spectral Type.	R.A. 1900·0	Dec. 1900·0	Proper Motion. R.A.	Proper Motion. Dec.	Parallax.	P.E.	No. of Plates.
		m		h m s		″	″	″	″	
	73° 767	8·1	Ko	17 22 58	+73° 6′	+ ·038	+·184	+·026	±·006	35
		9·0				+ ·050			±·005	
	68° 941	8·3	K2	17 35 47	+68° 52′	− ·069	+·128	+·009	±·009	30
		8·7				− ·076			±·006	
	35 Draco	5·1	F5	17 53 56	+76° 58′	+ ·048	+·239	+·039	±·013	27
	76° 667	5·4				+ ·099			±·010	
	79° 590	9·5		18 33 7	+79° 20′	+ ·053	+·356	+·029	±·008	25
		9·9				+ ·021			±·007	
	71° 933	8·8	Ko	19 9 21	+71° 9′	+ ·012	+·105	+·003	±·007	23
		9·3				− ·003			±·006	
	71° 956	7·5	Ko	19 31 14	+72° 3′	− ·068	+·110	−·003	±·008	22
		8·7				− ·080			±·007	
	67° 1162	8·0	Ko	19 31 33	+67° 29′	+ ·023	+·148	−·003	±·011	21
		8·3				+ ·033			±·009	
	L. 37923	8·4	Ko	19 42 11	+76° 11′	+ ·172	+·105	+·052	±·009	21
	76° 750	8·7				+ ·154	+·136		±·002	
	ε Draco	3·8	Ko	19 48 31	+70° 1′	+ ·081	+·031	+·030	±·016	27
	69° 1070	4·7				+ ·060			±·009	
	82° 607	8·6	Ko	20 7 53	+83° 2′	+ ·093	−·049	+·023	±·005	22
	1st star	10·1				+ ·040			±·004	
	82° 607			20 7 56		+ ·055	−·066	+·007	±·006	22
	2nd star	10·5			+83° 2′	+ ·032			±·005	
	ϰ Cephei	4·8	B9	20 12 16	+77° 25′	+ ·012	+·026	+·025	±·011	35
	77° 764	4·4				+ ·008			±·007	
	65° 1572	9·5		21 16 33	+65° 48′	+ ·182	+·166	+·013	±·007	25
		11·2				+ ·150			±·004	
	70° 1218	9·5		22 7 27	+70° 49′	+ ·245	−·096	−·012	±·011	20
		9·8				+ ·223			±·008	
	74° 985	9·0		22 46 12	+75° 6′	+ ·037	−·012	−·009	±·007	23
		9·2				+ ·006	−·005		±·002	
	74° 1018	8·0	Ko	23 18 14	+75° 14′	+ ·010	+·016	−·030	±·008	25
		8·4				+ ·050	+·008		±·003	

No.	Name and B.D. No.	Mag. Vis. Photog.	Spectral Type.	R.A. 1900·0.	Dec.	Proper Motion. R.A.	Proper Motion. Dec.	Parallax.	P.E.	No. of Plates.
		m		h m s		s	"	"	"	
49	69° 1329	9·0		23 20 53	+69° 24′	+ ·202	−·024	−·010	±·009	23
		9·3				+ ·201	−·030		±·002	
50	C. 3628	9·2		23 33 3	+83° 12′	+ ·187	+·025	·000	±·005	21
	82° 727	9·4				+ ·151	+·018		±·003	
51	68° 1395	9·2		23 42 21				−·028	±·006	19
	1st star	10·0			+68° 30′	− ·012	+·002		±·008	
52	68° 1395			23 42 23				−·015	±·006	17
	2nd star	10·3			+68° 30′	− ·016	+·007		±·008	

Notes.

Nos. 2, 47, 48, 51, 52.—Observed through being erroneously credited with large proper motions.

Nos. 5, 35, 41, 44.—A neutral-tinted filter employed in conjunction with the sector.

No. 12.—Principal star of Σ 638. Mags. 8·4, 10·1 (photog.).

No. 23.—Components not separated; measured as one mass.

Nos. 51, 52.—These form the double star 12558 of Burnham's General Catalogue.

Observed Colours of 205 *Red Stars.* By W. S. Franks.

The red stars have long claimed especial interest on account of their rich colouring, and because most of them are also variables. Various lists of such have been published, but they all suffer from the defect that the terms used to express their tints are too vague to be of much service. And when the estimates of several observers are collated, it is difficult to arrive at a satisfactory average value, owing to diversity of instruments and personal colour equation. Moreover, many stars commonly included in these lists are not "red" at all, but merely orange or orange yellow. A study of the *Draper Catalogue* prompted me to prepare a working list of the N, R, and S types therein contained that could be observed in this latitude, the limit being set at 25° S. declination. But other stars were met with, presumably of the same types according to their visual spectra, found by Dunér, Espin, and others, but not included in the "D.C." (probably on account of their faintness or variability); these were added to the working list. And a few advanced M-type stars, which were considered worthy of inclusion in a list of red stars, also found a place. During the work of observation (in 1923–4) any objects that turned out to be merely yellow, without any admixture of redness, were definitely rejected; this excluded several of the S-type stars, among others. It is believed that the resulting catalogue now

submitted is more truly a "red star" list than any hitherto published; being the work of one observer only, using the same telescope ($6\frac{1}{8}$-inch refractor) with which thousands of other colour determinations were made, and expressed in a definite nomenclature which allows the *mean* value to be readily deduced. As about two-thirds of the red stars are also variables, it may naturally be asked what effect the variation in brightness has upon the colour? Where the range is small, say about one or two magnitudes (which will include four-fifths of the N-type red variables), the effect is practically negligible. But where the range is large, say five or six magnitudes, as in the Md and S types, the tint is always paler near *maximum*, and deeper near *minimum*, than the normal mean. No stars of a pure red tint (like that of the spectrum) were ever met with; in the reddest of them an admixture of orange could always be detected. In type N the classes Na and Nb are practically indistinguishable visually, as already found by other observers; but class Nc is readily differentiated as a more intense tint. Both the R and S types appear to be intermediate between M and N, and may be collateral branches, as suggested by Merrill; their tints varying from orange or the faintest tinge of red up to the vivid hues of the N type. But there is an important difference between them in one respect—that whilst the S type is chiefly composed of large-range variables, the R type has scarcely any variables at all.

The following analysis will show how the tints are distributed in the various groups. The N-type stars are given according to their several classes, so far as could be ascertained; the remainder being set down as "unclassified." But objects belonging to types M, R, S, are dealt with in their broad groups.

The literal equivalents of the colour symbols used may be expressed thus:—OR^2 = *pale orange red*, $OR^{2\cdot5}$ = *medium orange red*, OR^3 = *full orange red*, $OR^{3\cdot5}$ = *intense orange red* In reducing the means, intermediate fractions in the exponent were sometimes unavoidable; these, for the purpose of analysis, were placed in the nearest adjacent column.

Spectra.	No. of Stars.	OR^2.	$OR^{2\cdot5}$.	OR^3.	$OR^{3\cdot5}$.	Kr-Ost. (Mean).	Fr (Mean).
M	21	10	10	1	...	7·83(10)	$OR^{2\cdot3}$*
R	20	13	6	1	...	8·10(11)	$OR^{2\cdot2}$
S	16	11	3	2	...	7·9(2)	$OR^{2\cdot2}$
Pec.	8	4	2	2	...	8·0(3)	$OR^{2\cdot4}$
Uncl. N	42	17	14	11	...	8·35(30)	$OR^{2\cdot4}$
Na	16	3	6	7	...	8·36(10)	$OR^{2\cdot6}$
Nb	63	12	30	21	...	8·34(36)	$OR^{2\cdot6}$
Nc	19	...	2	11	6	8·50(15)	$OR^{3\cdot1}$

Figures within parentheses are numbers of stars available from Kr-Ost.

* This value for the 21 M-type stars has no connection with the normal values for the three classes of M as given in *Monthly Notices*, **84**, 22, but merely relates to the reddest stars among the otherwise orange tints.

Catalogue of 205 Red Stars.

No.	Designation.	R.A. 1920.	Decl. 1920.	Mag.	Spect.	No. of Obs.	Mean Colour.
		h m	° ′				
1	SU Andromedæ	0 0·3	+43 10	var.	Na	7	$OR^{2\cdot5}$
2	ST Cassiopeiæ	,, 13·2	49 51	,,	Nb	6	$OR^{2\cdot5}$
3	VX Andromedæ	,, 15·7	44 16	,,	Nc	6	OR^{3}!
4	R ,,	,, 19·8	38 8	,,	S	6	$OR^{2\cdot5}$
5	+53°66 Cassiopeiæ	,, 20·1	53 49	9·3	R5	5	OR^{2}
6	* Es B 10 Andromedæ	,, 23·3	35 8	var.	Nb	7	$OR^{2\cdot8}$
7	+63°56 Cassiopeiæ	,, 26·5	63 25	9·0	N ?	5	OR^{2}
8	U ,,	,, 41·9	47 49	var.	S	5	OR^{2}
9	W ,,	,, 50·2	58 8	,,	Mbp	5	OR^{2}
10	Z Piscium	1 11·7	25 14	,,	Na	7	$OR^{2\cdot5}$
11	S Cassiopeiæ	,, 13·7	72 10	,,	S	5	$OR^{2\cdot5}$
12	Anon. Cassiopeiæ	,, 39·0	60 13	9·3	N ?	5	$OR^{2\cdot5}$
13	+53°379 Persei	,, 40·0	53 34	9·2	R5	5	$OR^{2\cdot5}$
14	X Cassiopeiæ	,, 52·9	58 53	var.	Md ?	5	OR^{2}
15	V Arietis	2 10·7	11 53	,,	Nb	6	OR^{3}
16	−10°513 Ceti	,, 31·2	− 9 48	8·0	R3	5	OR^{2}
17	UY Andromedæ	,, 33·3	+38 49	var.	Nb	6	$OR^{2\cdot5}$
18	+58°501 Persei	,, 33·8	59 15	,,	N ?	5	$OR^{2\cdot5}$
19	Es B 64 ,,	3 5·2	57 36	8·0	R5	5	$OR^{2\cdot2}$
20	,, 66 ,,	,, 8·1	47 32	var.	Nc	5	OR^{3}!
21	Y ,,	,, 22·2	43 54	,,	Nb	6	$OR^{2\cdot5}$
22	U Camelopardi	,, 34·9	62 33	,,	Nb	6	OR^{3}
23	Es B 81 Camelopardi	,, 58·9	61 35	7·9	R8	5	$OR^{2\cdot5}$
24	SY Persei	4 10·5	50 26	var.	Nc ?	5	OR^{3}!
25	T Camelopardi	,, 32·4	66 2	,,	S	5	OR^{2}
26	Anon. Persei	,, 33·9	41 28	,,	Nc ?	5	OR^{3}
27	ST Camelopardi	,, 42·8	68 2	,,	Nb	7	OR^{3}
28	Es B 98 Aurigæ	,, 44·0	34 52	9·0	Nb	6	OR^{2}
29	,, 100 Tauri	,, 46·0	15 39	9·4	Na	6	$OR^{2\cdot5}$
30	TT ,,	,, 46·5	28 24	var.	Nb	6	OR^{3}
31	Es B 102 Aurigæ	,, 47·0	38 22	8·5	N ?	6	$OR^{2\cdot8}$
32	R Orionis	,, 54·6	8 1	var.	S	4	OR^{2}
33	R Leporis	,, 56·0	−14 56	,,	Nc	7	$OR^{3\cdot5}$
34	+50°1112 Aurigæ	,, 57·1	+50 31	,,	Nb	5	OR^{3}
35	+38°1010 ,,	,, 57·8	38 57	9·2	N ?	5	$OR^{2\cdot5}$
36	W Orionis	5 1·2	1 3	var.	Nb	6	OR^{3}
37	T Leporis	,, 1·4	−22 1	,,	Md	4	OR^{2}
38	TX Aurigæ	,, 3·6	+38 54	,,	Nb	5	OR^{3}

* Es B = Birmingham's Red Star Catalogue, revised by Espin, 1890.

Catalogue of 205 Red Stars—continued.

No.	Designation.	R.A. 1920.	Decl. 1920.	Mag.	Spect.	No. of Obs.	Mean Colour.
		h m	° ′				
39	Es B 117 Eridani	5 5·9	− 5 38	8·5	Nb	6	OR^{2}
40	R Aurigæ	„ 10·8	+53 30	var.	Mc	5	OR$^{2·5}$
41	Es B 125 Aurigæ	„ 13·8	35 43	8·9	Nb	5	OR^{2}
42	UV „	„ 16·6	32 26	var.	Np	5	OR$^{2·5}$
43	S „	„ 21·8	34 5	„	Nb	6	OR$^{2·5}$
44	S Orionis	„ 25·1	− 4 45	„	Mc	5	OR^{2}
45	RT „	„ 28·9	+ 7 5	„	Nb	5	OR$^{2·3}$
46	−25°2539 Leporis	„ 32·5	−25 47	7·9	Na	5	OR$^{2·5}$
47	S Camelopardi	„ 32·5	+68 46	var.	R8	5	OR$^{2·5}$
48	Es B 148 Tauri	„ 33·6	24 57	9·4	Mc	5	OR^{2}
49	TU „	„ 40·3	24 23	var.	Nb	5	OR^{3}
50	Y „	„ 40·9	20 39	„	Nb	5	OR^{3}
51	Es B 156 Aurigæ	„ 42·7	44 49	9·2	Nc	5	OR^{3}
52	„ 158 „	„ 43·0	30 35	var.	Na	6	OR$^{2·5}$
53	U Orionis	„ 51·1	20 10	„	Map	6	OR$^{2·5}$
54	TU Geminorum	6 5·9	26 2	„	Na	7	OR^{3}
55	+29°1177 Geminorum	„ 14·5	29 30	9·5	N ?	5	OR$^{2·5}$
56	V Aurigæ	„ 18·0	47 43	var.	Nb	5	OR$^{2·5}$
57	+3°1214 Monocerotis	„ 18·2	3 28	8·9	Mcp	5	OR$^{2·5}$
58	Es B 184 Geminorum	„ 19·1	25 3	var.	Nb	6	OR^{3}
59	„ 187 „	„ 20·9	14 46	„	Nb	7	OR$^{2·5}$
60	„ 189 „	„ 21·5	19 9	„	Nc	5	OR^{3}
61	RV Aurigæ	„ 29·0	42 33	„	Na	6	OR$^{2·5}$
62	UU „	„ 31·0	38 31	„	Na	7	OR^{3}
63	Anon. Geminorum	„ 34·5	22 41	9·2	Nc	5	OR$^{2·5}$
64	Es 564 „	„ 37·0	31 32	var.	Na	5	OR$^{2·5}$
65	Es B 200 Monocerotis	„ 40·5	3 24	9·3	Nc	5	OR^{3}
66	„ 201 „	„ 43·4	0 47	9·6	Nc	5	OR$^{2·5}$
67	„ 206 „	„ 49·2	− 4 28	var.	Nb	5	OR^{2}
68	RV „	„ 54·1	+ 6 16	„	Nb	6	OR$^{2·5}$
69	R Lyncis	„ 54·7	55 27	„	S	5	OR^{2}
70	Es B 211 Monocerotis	„ 57·0	− 3 8	7·7	R5	5	OR$^{2·3}$
71	Anon. Camelopardi	„ 57·5	+68 19	9·6	Nc	5	OR^{3} !
72	R Geminorum	7 2·5	22 50	var.	S	5	OR^{2}
73	Es B 216 Monocerotis	„ 3·1	− 7 26	8·3	Nc	5	OR^{3} !
74	W Canis Majoris	„ 4·3	−11 46	var.	Na	6	OR^{3}
75	R „ Minoris	„ 4·3	+10 9	„	S	6	OR^{3}
76	Es B 225 Geminorum	„ 16·0	25 9	„	Nb	5	OR$^{2·5}$

Catalogue of 205 *Red Stars*—continued.

No.	Designation.	R.A. 1920.	Decl. 1920.	Mag.	Spect.	No. of Obs.	Mean Colour.
		h m	° ′				
77	−3°1873 Monocerotis	7 19·1	− 4 3	9·0	Nb	5	$OR^{2\cdot8}$
78	−22°1850 Puppis	,, 20·3	−22 49	9·2	Nb	6	$OR^{2\cdot5}$
79	Es B 230 Monocerotis	,, 21·2	− 2 59	var.	Nb	5	OR^{2}
80	−11°1941 ,,	,, 23·4	−11 33	9·0	S	5	OR^{2}
81	Es B 235 Geminorum	,, 27·1	+24 36	8·0	R8	5	OR^{2}
82	,, 240 Canis Minoris	,, 32·3	2 15	var.	Na	5	$OR^{2\cdot8}$
83	W ,,	,, 44·5	5 36	,,	Na	5	$OR^{2\cdot5}$
84	T Geminorum	,, 44·5	23 56	,,	S	5	OR^{2}
85	−18°2040 Puppis	,, 46·2	−18 48	8·5	S	4	OR^{2}
86	RU ,,	8 4·1	−22 40	var.	Nb	5	OR^{2}
87	RY Hydræ	,, 15·9	+ 3 1	,,	Nb	5	$OR^{2\cdot5}$
88	−17°2442 Puppis	,, 16·1	−18 1	9·3	R5	5	$OR^{2\cdot5}$
89	X Cancri	,, 50·9	+17 32	var.	Nb	6	$OR^{2\cdot7}$
90	T ,,	,, 52·1	20 9	,,	N ?	5	OR^{2}
91	+34°1929 Cancri	,, 54·8	34 4	9·2	Ro	5	$OR^{1\cdot5}$
92	+50°1603 Ursæ Majoris	,, 57·6	50 24	9·5	R5	5	$OR^{2\cdot5}$
93	+14°2048 Leonis	9 9·4	14 32	8·6	R5	5	OR^{2}
94	R Leonis Minoris	,, 40·8	34 53	var.	Mb	6	$OR^{2\cdot5}$
95	R Leonis	,, 43·3	11 48	,,	Mc	6	OR^{3}
96	Anon. Ursæ Majoris	,, 45·7	53 1	9·5	R3	5	OR^{2}
97	−1°2312 Sextantis	,, 46·9	− 1 39	var.	Nb	5	OR^{2}
98	Y Hydræ	,, 47·4	−22 38	,,	Nap	7	$OR^{2\cdot2}$
99	U ,,	10 33·6	−12 58	,,	Nb	6	OR^{3}
100	Es B 318 Ursæ Majoris	,, 39·5	+67 50	6·3	Na	5	OR^{3}
101	V Hydræ	,, 47·7	−20 50	var.	N ?	5	OR^{2}
102	R Crateris	,, 56·6	−17 53	,,	Mc	6	$OR^{2\cdot5}$
103	−13°3407 Crateris	11 31·7	−14 9	8·7	Ro	6	OR^{2}
104	SS Virginis	12 21·1	+ 1 13	var.	Nc	6	$OR^{3\cdot3}$
105	S Ursæ Majoris	,, 40·4	61 31	,,	S	5	$OR^{2\cdot5}$
106	Y Canum Venaticorum	,, 41·4	45 52	,,	Nb	6	$OR^{2\cdot8}$
107	RY Draconis	,, 53·3	66 25	,,	Np	5	OR^{3}
108	+38°2389 Canum Ven.	,, 55·8	38 15	,,	Na	5	$OR^{2\cdot5}$
109	SW Virginis	13 9·9	− 2 22	,,	Mc	5	OR^{2}
110	W ,,	,, 21·9	− 2 58	,,	Np	5	OR^{2}
111	R Hydræ	,, 25·3	−22 52	,,	Mc	6	$OR^{2\cdot5}$
112	−17°3961 Hydræ	,, 51·4	−17 51	8·1	S	4	$OR^{1\cdot5}$
113	RX Boötis	14 20·6	+26 5	var.	Mc	6	OR^{2}
114	R Camelopardi	,, 23·4	84 12	,,	S	6	OR^{2}

Catalogue of 205 Red Stars—continued.

No.	Designation.	R.A. 1920.	Decl. 1920.	Mag.	Spect.	No. of Obs.	Mean Colour.
		h m	° ′				
115	V Coronæ Borealis	15 46·7	+39 49	var.	Nb	5	$OR^{2\cdot5}$
116	X Herculis	16 0·3	47 27	„	Mc	6	OR^{2}
117	V Ophiuchi	„ 22·2	−12 15	„	Nb	5	$OR^{2\cdot5}$
118	TW „	17 25·0	−19 25	7·8	Nb	5	$OR^{2\cdot5}$
119	SZ Sagittarii	„ 40·2	−18 37	var.	Nb	5	$OR^{2\cdot3}$
120	T Draconis	„ 55·1	+58 14	„	N ?	5	OR^{2}
121	+9°3576 Aquilæ	18 5·0	9 26	9·2	R_5	4	$OR^{2\cdot3}$
122	−15°4923 Sagittarii	„ 14·7	−15 39	var.	R_5	4	OR^{2}
123	TY Ophiuchi	„ 27·4	+ 4 20	„	Nb	5	$OR^{2\cdot5}$
124	T Lyræ	„ 29·6	36 56	„	Nc	6	$OR^{3\cdot3}$
125	RX Scuti	„ 32·8	− 7 40	„	Nb	5	$OR^{2\cdot5}$
126	+6°3898 Aquilæ	„ 38·0	+ 6 44	9·1	S	4	OR^{3}
127	RV Scuti	„ 39·9	−13 18	var.	Nb	4	$OR^{2\cdot5}$
128	Es B 561 Lyræ	„ 40·1	+36 53	„	Nb	6	$OR^{2\cdot5}$
129	S Scuti	„ 46·4	− 8 0	„	Nb	6	OR^{3}
130	T „	„ 51·5	− 8 18	„	Nb	5	$OR^{2\cdot5}$
131	UW Aquilæ	„ 53·4	+ 0 21	9·1	N ?	5	$OR^{2\cdot5}$
132	UV „	„ 54·9	14 15	var.	Nb	5	$OR^{2\cdot5}$
133	+10°3764 Aquilæ	„ 58·5	10 7	9·2	N ?	5	$OR^{2\cdot5}$
134	V „	19 0·1	− 5 48	var.	Nc	7	$OR^{3\cdot5}$
135	R „	„ 2·5	+ 8 6	„	Mc	5	OR^{2}
136	Anon. Vulpeculæ	„ 10·4	27 10	9·3	N ?	5	OR^{3}
137	Es B 595 Sagittarii	„ 14·6	−16 4	var.	Na	5	$OR^{2\cdot8}$
138	+10°3857 Aquilæ	„ 14·7	+10 22	9·0	N ?	5	OR^{2}
139	U Lyræ	„ 17·3	37 44	var.	Md	4	OR^{2}
140	T Sagittæ	„ 18·1	17 30	„	Mc	4	$OR^{2\cdot5}$
141	−10°5057 Aquilæ	„ 18·6	−10 52	7·0	Ro	4	$OR^{1\cdot7}$
142	Es B 607 Draconis	„ 24·4	+76 25	6·5	Nb	6	$OR^{2\cdot5}$
143	AQ Sagittarii	„ 29·4	−16 33	var.	Nb	5	OR^{2}
144	R Cygni	„ 34·8	+50 1	„	S	5	OR^{2}
145	TT „	„ 37·9	32 26	„	Nb	5	$OR^{2\cdot3}$
146	Anon. Sagittæ	„ 43·7	15 52	9·2	N ?	5	$OR^{2\cdot2}$
147	χ^2 Cygni	„ 47·5	32 43	var.	Map	6	$OR^{2\cdot5}$
148	AX „	„ 54·6	44 2	„	Nb	6	OR^{3}
149	Anon. Aquilæ	„ 57·3	9 17	8·7	Nb	5	$OR^{2\cdot2}$
150	„ Cygni	„ 58·0	30 40	9·2	N ?	4	OR^{2}
151	„ „	„ 58·9	39 48	9·1	N ?	4	$OR^{2\cdot5}$
152	Es B 639a Sagittæ	„ 59·3	20 50	var.	Nb	4	OR^{2}

Catalogue of 205 *Red Stars*—continued.

No.	Designation.	R.A. 1920. h m	Decl. 1920. ° ′	Mag.	Spect.	No. of Obs.	Mean Colour.
153	Anon. Delphini	20 1·2	+ 9 7	8·6	N ?	4	OR^{2}
154	X Sagittæ	,, 1·6	20 26	var.	Nb	4	OR^{3}
155	AA Cygni	,, 1·6	36 34	,,	Np	4	OR^{2}
156	AY ,,	,, 7·0	41 16	,,	N ?	4	$OR^{2·5}$
157	SV ,,	,, 7·3	47 37	,,	Nb	5	$OR^{2·5}$
158	RY ,,	,, 7·4	35 42	,,	Nb	5	OR^{3}
159	RS ,,	,, 10·6	38 29	,,	Np	5	OR^{3}
160	RT Capricorni	,, 12·4	−21 34	,,	Nb	6	$OR^{2·8}$
161	Anon. Cygni	,, 12·5	+36 50	9·6	N ?	5	OR^{3}
162	SZ Cephei	,, 12·9	76 56	var.	Pec.	4	OR^{2}
163	Es B 659a Cygni	,, 13·8	49 41	,,	N ?	4	$OR^{2·3}$
164	WX ,,	,, 15·6	37 8	,,	Nb	5	$OR^{2·5}$
165	U ,,	,, 17·0	47 38	,,	R8	5	OR^{3} !
166	Es 458 ,,	,, 18·1	35 21	,,	N ?	4	OR^{2}
167	Es B 664 ,,	,, 18·4	36 40	,,	Mc ?	5	$OR^{2·5}$
168	,, 665 ,,	,, 18·7	37 17	,,	N ?	5	$OR^{2·8}$
169	−12°5755 Aquarii	,, 27·3	−12 9	9·0	R5	4	$OR^{1·8}$
170	AD Cygni	,, 28·2	+32 18	var.	Np	4	$OR^{2·5}$
171	+40°4210 Cygni	,, 28·2	40 15	9·2	N ?	4	OR^{2}
172	V ,,	,, 38·7	47 51	var.	N ?	5	OR^{3}
173	Es B 688 ,,	,, 44·0	45 45	8·8	Pec.	5	OR^{2}
174	+32°3954 ,,	,, 46·0	32 55	9·2	N ?	4	$OR^{2·7}$
175	Anon. ,,	,, 56·6	43 38	9·2	N ?	4	$OR^{2·5}$
176	,, ,,	21 7·3	33 2	9·1	N ?	5	$OR^{2·1}$
177	,, Cephei	,, 13·2	53 58	9·2	N ?	5	OR^{2}
178	,, Cygni	,, 15·0	51 54	9·5	N ?	6	$OR^{2·2}$
179	+49°3500 Cygni	,, 16·1	49 23	9·3	N ?	6	$OR^{2·5}$
180	YY ,,	,, 19·3	42 3	var.	Nc	6	OR^{3} !
181	+49°3535 ,,	,, 26·4	49 59	9·3	N ?	5	$OR^{2·2}$
182	+47°3429 ,,	,, 26·7	48 12	9·2	N ?	5	$OR^{2·1}$
183	+60°2267 Cephei	,, 33·0	60 32	9·3	Nb	5	OR^{3}
184	S Cephei	,, 36·2	78 16	var.	Nc	6	$OR^{3·4}$
185	Es B 711 Cygni	,, 38·6	35 8	6·4	Nb	5	$OR^{2·5}$
186	RV ,,	,, 40·0	37 39	var.	Ncp	7	OR^{3} !
187	Anon. Cephei	,, 43·7	52 11	9·3	N ?	5	OR^{2}
188	+52°3036 Cephei	,, 44·3	52 18	9·0	N ?	5	$OR^{2·1}$
189	+53°2736 ,,	,, 52·2	54 6	8·6	N ?	5	$OR^{2·7}$
190	+49°3673 ,,	,, 52·3	50 7	var.	Nc	5	$OR^{3·2}$

Catalogue of 205 *Red Stars*—continued.

No.	Designation.	R.A. 1920.	Decl. 1920.	Mag.	Spect.	No. of Obs.	Mean Colour.
		h m	° ′				
191	RX Pegasi	21 52·6	+22 30	var.	Nb	5	OR$^{2\cdot5}$
192	Anon. Cephei	22 24·7	64 23	9·6	N ?	5	OR$^{2\cdot7}$
193	+60°2432 Cephei	,, 41·2	61 18	8·6	Nb	6	OR$^{2\cdot7}$
194	+53°3033 ,,	,, 52·6	53 47	9·4	Nb	3	OR$^{2\cdot5}$
195	UY Andromedæ	,, 58·2	45 27	var.	N ?	6	OR$^{2\cdot4}$
196	Anon. Cassiopeiæ	23 6·8	60 49	9·0	N ?	6	OR$^{2\cdot2}$
197	,, Andromedæ	,, 19·5	44 57	var.	N ?	5	OR$^{2\cdot8}$
198	+48°4048 Andromedæ	,, 23·1	49 4	,,	Nb	6	OR$^{2\cdot5}$
199	ST ,,	,, 34·8	35 50	,,	Nb	6	OR$^{2\cdot3}$
200	19 Piscium	,, 42·3	3 2	5·3	Na	7	OR^{3}
201	+5°5223 Piscium	,, 45·0	5 57	8·8	R$_3$	6	OR^{2}
202	TZ Cassiopeiæ	,, 49·0	60 33	var.	N ?	6	OR$^{2\cdot8}$
203	R ,,	,, 54·3	50 56	,,	Mc	5	OR$^{2\cdot6}$
204	+56°3126 Cassiopeiæ	,, 55·1	56 31	9·0	N ?	6	OR^{3}
205	+59°2810 ,,	,, 57·2	59 55	var.	Nb	5	OR$^{2\cdot2}$

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New Red Stars. By Rev. T. E. Espin.

The following list contains red stars noted in the course of the search for new double stars. The observations prior to 1914 September were made with the 17¼-inch, after that date with the 24-inch.

No.	B.D.	R.A. 1900.	Decl.	Mags. B.D.	Mags. Obs.	Col.	Spect.*	Date.	
	°	h m	° ′					1900+	
1	..	0 2·2	+43 21	..	9·4	R	IV.	16 Jan. 9	Var. ?
2	+45·2	3·6	45 20	9·3	9·0	R	III. ?	13 Nov. 17	
3	+42·35	9·7	42 28	9·5	9·3	OR	III. !!	16 Nov. 26	
4	+42·43	11·3	42 56	9·1	9·2	R	III. ?	16 Oct. 25	
5	+43·69	18·1	43 29	9·2	8·6	OR	III. !	16 Oct. 18	
6	+57·80	20·9	57 25	8·5	8·5	R	III. !!	18 Nov. 6	
7	+47·128	27·5	48 6	9·5	9·2	R	..	13 Nov. 17	
8	+43·118	31·9	43 50	9·5	9·3	R	..	15 Nov. 17	
9	+47·156	33·3	47 36	9·0	9·0	R	..	13 Nov. 17	
10	+57·144	41·5	57 56	9·3	8·7	R	III.	19 Jan. 18	

* The marks of expression (!, !!, !!!) refer to the intensity of the bands.

No.	B.D.	R.A. 1900.	Decl.	Mags. B.D.	Mags. Obs.	Col.	Spect.	Date.	
	°	h m	° ′					1900+	
11	+59·130	0 45·4	+59 18	9·1	9·0	OR	III. !	18 Nov. 8	
12	+60·125	47·1	60 32	9·2	9·0	R	III. ?	21 Dec. 3	
13	+37·174	51·1	37 51	8·9	9·0	R	III. !!	22 Nov. 15	
14	+60·152	56·8	60 26	9·1	8·8	OR	III. !!	21 Dec. 3	
15	+57·221	1 7·1	58 8	8·7	8·9	OR′	III.	18 Nov. 5	
16	..	18·1	45 29	..	9·1	R	..	14 Nov. 28	
17	..	18·8	60 19	..	9·2	R	III. ?	21 Nov. 7	
18	+37·286	22·1	37 26	9·1	9·2	OR	III. !!	22 Dec. 15	
19	+60·287	31·3	60 32	9·0	9·0	R	..	21 Nov. 10	
20	+42·431	55·5	43 1	8·8	8·7	OR	III. ?	16 Oct. 25	
21	+51·504	2 2·0	51 19	9·3	9·1	R	III. !!	09 Dec. 31	
22	..	43·0	48 44	..	9·2	R	III. !!	12 Oct. 16	
23	+57·704	3 4·3	57 18	9·1	9·0	OR	III. !	18 Nov. 11	
24	+61·588	20·2	61 13	9·5	9·3	OR	III. !!	21 Jan. 10	
25	+43·559	36·4	44 6	9·4	9·2	R	..	15 Jan. 25	
26	+44·788	38·1	44 28	9·5	8·9	RR	III. !!	15 Jan. 17	Var. ?
27	..	39·7	52 22	..	var.	R	III.	09 Dec. 18	
28	..	47·4	62 17	..	9·2	R	III. !!	21 Jan. 10	
29	..	49·6	61 39	..	9·2	R	..	22 Feb. 1	
30	..	4 5·6	40 4	..	11·8	R	IV.	15 Jan. 9	Note.
31	+40·917	9·4	40 9	9·5	9·4	R	IV. ?	15 Jan. 9	
32	+42·955	17·9	42 8	8·9	8·8	OR	III.	16 Dec. 26	
33	..	27·4	42 29	..	9·2	R	III. !!	16 Dec. 26	
34	+44·1017	39·4	44 32	9·5	9·1	OR	III. !	15 Jan. 17	
35	..	5 45·8	60 12	..	9·2	OR	III. !	21 Jan. 6	
36	+59·1024	6 42·9	59 25	8·5	8·0	R	III. !!	19 Feb. 18	
37	+61·924	52·3	61 11	9·5	9·4	R	III. !!	21 Jan. 6	
38	+44·1590	7 2·6	44 22	9·5	9·4	OR	III. !	15 Feb. 15	
39	..	11·6	38 15	..	8·8	OR	III. !!	24 Feb. 29	
40	+45·1497	45·0	45 32	9·4	8·8	R	IV. ?	14 Mar. 6	Var. ?
41	+44·1717	8 2·5	44 42	9·5	9·0	OR	III. !	15 Feb. 15	
42	+56·1458	10 24·1	56 30	8·5	9·2	R	III. ?	19 Mar. 3	
43	+46·1635	25·5	46 2	9·3	8·4	OR′	III. !	13 Apr. 2	Var. ?
44	+44·2267	13 17·0	44 30	9·3	9·4	R	IV. ?	13 May 13	
45	+40·3130	17 15·5	40 54	9·0	9·1	R	III. ?	16 Aug. 3	
46	+40·3151	24·6	40 21	7·7	7·8	OR	II.–III.	16 Aug. 10	
47	+40·3150	24·4	40 11	9·0	8·5	R	III. !	16 Aug. 20	
48	+40·3160	28·0	40 34	9·0	8·9	R	III. ?	16 Aug. 8	
49	+40·3189	35·7	40 57	7·5	7·9	OR	III. !	16 Aug. 8	

No.	B.D.	R.A. 1900.	Decl.	Mags. B.D.	Mags. Obs.	Col.	Spect.	Date. 1900+	
	°	h m	° ′						
50	+59·1864	17 54·4	+59 6	9·3	8·7	OR	III.	18 Sept. 18	Var. ?
51	+41·2965	18 0·7	41 21	7·5	8·2	OR	III. !!	16 Aug. 30	
52	+56·2078	12·2	56 43	8·9	8·4	OR	III. !!	18 Sept. 30	
53	+44·2267	16·8	44 29	9·3	9·2	R	IV. ?	12 May 13	
54	+41·3051	22·3	41 15	8·7	8·5	OR′	III. !	16 Aug. 30	
55	+58·1818	26·1	58 25	9·3	9·4	R	III. !!	19 Aug. 20	
56	+58·1819	26·7	58 29	8·7	9·0	OR	III. !	19 Aug. 20	
57	+39·3447	28·5	39 22	9·2	9·2	OR	III. !!	16 Sept. 4	
58	+39·3465	31·9	39 40	9·4	9·5	OR	III. ?	16 Sept. 4	
59	+39·3475	34·8	39 30	8·8	9·3	OR	III. !	16 Sept. 4	
60	+39·3483	36·4	39 22	8·5	9·0	R	III. !!	16 Sept. 4	
61	+39·3482	36·4	39 49	9·2	9·4	R	III. ?	16 Sept. 4	
62	+59·1908	36·4	59 41	8·5	8·0	OR	III. !	18 Sept. 27	
63	+63·1440	37·0	63 48	9·3	9·0	R	III. ?	21 Sept. 25	
64	+39·3497	38·6	39 56	9·1	9·1	OR	III. !	16 Sept. 4	
65	+37·3223	39·4	37 21	8·2	8·5	OR	III. !	16 Sept. 18	
66	+39·3503	39·5	39 50	9·0	9·3	R	III. !!!	16 Sept. 4	
67	+37·3229	40·9	37 57	8·7	8·5	OR	III.	16 Sept. 18	
68	+44·2987	42·4	44 57	9·0	9·0	R	III. !!	13 Aug. 25	
69	+56·2133	43·2	56 29	9·1	9·0	OR	III. !!	18 Sept. 11	
70	+41·3145	44·7	41 39	9·0	8·5	R	III. !!	16 Sept. 24	
71	+39·3555	48·8	39 12	8·8	9·0	R	III. !	16 Sept. 16	
72	+37·3274	50·3	37 54	9·5	9·5	R	III. !!	16 Sept. 4	
73	+57·1920	54·2	57 31	8·6	8·8	OR′	III. !	18 Sept. 13	
74	+44·3032	56·5	44 59	9·2	8·9	R′	III. !!	13 Nov. 5	
75	..	57·5	61 56	..	9·0	R	III. !!	20 Sept. 19	
76	+39·3608	57·6	39 38	9·5	9·3	R	III. !!	16 Sept. 1	
77	+36·3388	19 0·2	36 46	9·4	9·3	R	III. !!	16 Sept. 4	
78	+59·1937	0·7	59 59	7·0	7·2	OR	III. !	18 Oct. 4	
79	+41·3244	4·9	41 57	8·8	9·0	OR	III. !	16 Aug. 10	
80	+56·2196	7·4	56 24	9·0	8·5	OR	III.	18 Oct. 23	
81	..	9·0	57 3	..	9·0	R	III. !!	18 Sept. 30	Var. ?
82	+46·2668	17·3	46 44	8·3	8·5	OR′	III. ?	13 Nov. 7	
83	..	21·3	46 10	..	9·3	R	III. !	13 Nov. 5	
84	+59·2030	23·5	59 34	8·2	8·0	OR	III.	18 Sept. 27	
85	+41·3356	24·5	41 46	9·0	8·5	OR	III. !!	16 Sept. 24	
86	+41·3358	24·7	41 48	8·9	8·6	OR	III. !	16 Sept. 24	
87	+43·3274	29·1	43 17	9·3	9·2	OR	III. !!	15 Sept. 30	
88	+41·3410	32·5	42 2	8·7	8·1	OR	III.	16 Sept. 13	

No.	B.D.	R.A. 1900.	Decl.	Mags. B.D.	Mags. Obs.	Col.	Spect.	Date. 1900+	
	°	h m	° ′						
89	+42·3393	19 32·6	+42 8	8·4	7·7	OR′	III.	16 Sept. 13	
90	+44·3183	32·8	44 39	8·9	8·3	OR	III. !!	13 Nov. 5	
91	+42·3404	34·8	42 19	9·0	8·5	OR′	III. ?	16 Sept. 13	
92	+42·3405	34·9	42 37	9·0	8·6	R	III. !!	16 Aug. 10	
93	+42·3407	35·0	42 12	8·9	8·2	OR	III.	16 Sept. 13	
94	..	38·2	39 56	..	9·4	RR	III. !!!	16 Sept. 4	
95	+42·3426	39·9	42 17	9·5	9·4	R	III. !!	16 Aug. 30	
96	+44·3223	40·4	44 23	9·5	9·6	R	III. !!	14 Nov. 9	
97	+55·2250	41·4	55 23	8·6	8·9	OR	III. !	19 Oct. 14	
98	+42·3444	42·1	42 17	9·1	8·6	OR	III. !!	16 Sept. 16	
99	+44·3247	44·0	44 41	9·5	9·3	R	III. !!!	14 Nov. 9	
100	+41·3490	44·4	42 4	7·9	8·0	OR′	..	16 Oct. 26	
101	+46·2784	45·0	46 54	9·4	9·3	R′	III. !	13 Nov. 7	
102	..	48·1	47 3	..	9·5	R	III. !	13 Nov. 7	
103	+47·2939	48·7	47 8	9·3	9·4	R	III.	13 Nov. 7	
104	+42·3503	49·3	42 23	9·5	9·3	OR	?	16 Sept. 24	
105	+43·3418	53·4	43 33	9·5	9·3	OR	III. ?	13 Nov. 3	
106	+45·3016	53·8	45 37	9·3	9·5	OR	III. !	13 Aug. 25	
107	+42·3541	54·2	42 47	8·9	8·6	OR	III. !	16 Sept. 13	
108	+42·3544	54·8	42 15	8·5	8·4	R	III. !	16 Oct. 8	
109	+41·3562	56·9	41 57	7·0	7·7	OR′	II.	16 Oct. 26	
110	+41·3563	57·0	41 54	9·0	9·0	R	III. !	16 Oct. 26	
111	+42·3581	20 1·5	42 56	8·7	8·5	OR	III.	15 Nov. 16	
112	+57·2126	1·5	57 16	9·0	8·5	OR	III. !	18 Sept. 28	
113	+44·3337	2·0	44 51	9·4	9·4	OR′	III. ?	14 Nov. 9	
114	+44·3342	3·3	44 17	9·5	9·5	R	III. !!	14 Nov. 9	
115	..	9·0	47 29	..	9·5	R	III. !!	13 Nov. 7	
116	+45·3097	9·7	45 30	9·5	9·2	R	III. !!	13 Aug. 24	
117	..	14·9	42 33	..	9·3	R	III. !!	16 Sept. 13	
118	+42·3684	15·3	42 18	9·0	8·8	R	III. !!!	16 Sept. 24	
119	..	17·8	35 15	..	9·7	OR	IV. ?	14 Sept. 19	
120	+42·3712	19·0	42 22	9·5	9·5	R	III. !!	16 Oct. 16	
121	+39·4170	20·9	39 21	8·8	8·5	OR′	III.	22 Oct. 28	
122	..	46·9	45 2	..	9·3	R	IV.	14 Sept. 15	Var.
123	+43·3770	53·4	43 33	9·3	9·0	R	III. !!	14 Nov. 28	
124	+42·2922	54·3	42 57	9·5	9·4	R	III. !!	15 Dec. 8	
125	+58·2192	54·5	58 27	9·0	9·2	R	III. !!	18 Sept. 13	
126	..	57·9	44 24	..	9·7	R	IV. ?	14 Sept. 17	
127	+59·2312	21 1·1	59 46	8·8	8·5	OR	III. !!	18 Nov. 8	

No.	B.D.	R.A. 1900.	Decl.	Mags. B.D.	Mags. Obs.	Col.	Spect.	Date 1900+.	
	°	h m	° ′						
128	..	21 4·9	+45 3	..	9·7	R	IV. ?	13 Oct. 31	Var.
129	+41·4035	8·3	42 11	9·4	9·3	R	IV. ?	16 Oct. 21	
130	+55·2545	11·8	55 45	9·0	8·7	R	III. !!	19 Oct. 14	
131	+43·3860	11·9	43 29	9·5	9·5	R	III. !	14 Nov. 28	
132	+59·2357	17·7	60 7	9·0	9·0	R	III. !!!	18 Nov. 8	
133	+41·4116	18·6	41 52	9·5	9·5	OR	III. !!	16 Aug. 30	
134	+45·3526	20·8	45 30	8·8	8·9	OR	III.	13 Nov. 29	
135	+42·4091	21·2	42 25	8·9	8·8	OR	III.	16 Oct. 25	
136	+45·3531	21·7	45 37	8·0	8·0	OR′	III. ?	13 Nov. 29	
137	..	38·0	41 56	..	9·5	R	III. !!	16 Aug. 30	
138	..	38·0	44 11	..	9·1	R	III. ?	14 Nov. 27	
139	+37·4401	38·6	37 58	8·8	..	OR	III. !	22 Nov. 13	
140	..	39·2	37 51	..	10·7	R	III.	22 Nov. 13	
141	+41·4248	39·9	41 39	9·4	9·1	R	III. !!!	16 Sept. 13	Var.
142	+41·4287	47·1	41 49	9·3	9·3	OR	III.	16 Sept. 1	
143	..	51·5	45 40	..	9·5	R′	III. ?	13 Nov. 7	
144	+59·2433	52·4	60 9	9·5	9·2	OR	III. ?	18 Nov. 8	
145	+41·3937	52·7	41 51	9·0	8·4	OR	III. !	16 Sept. 23	
146	+41·4343	55·9	41 16	9·3	9·3	R	III. !!	16 Sept. 23	
147	..	59·5	45 5	..	9·3	R	IV. !!	14 Sept. 17	Var. ?
148	+41·4361	59·5	41 25	9·0	8·5	OR	III. !	16 Sept. 1	
149	+60·2333	22 1·6	60 52	8·2	8·9	R	III. !	21 Oct. 23	
150	+62·2030	2·1	62 36	9·0	8·7	R	III. !!!	21 Nov. 2	
151	+62·2031	2·3	62 14	9·5	9·4	R	III. !!!	21 Nov. 6	
152	+40·4731	4·0	41 7	9·4	9·5	OR	III. ?	16 Sept. 23	
153	+41·4413	7·4	41 26	9·0	8·6	OR	III. !!	16 Sept. 5	
154	+41·4418	8·0	41 34	9·1	8·3	OR	III.	16 Sept. 5	
155	+40·4764	9·8	41 1	9·4	8·5	OR	III. !	16 Oct. 16	
156	+40·4765	10·1	41 5	9·3	8·7	OR	III. !!	16 Oct. 16	
157	+41·4439	11·5	41 30	9·3	9·2	R	III.	16 Oct. 16	
158	+41·4456	14·5	41 38	7·8	8·2	OR	III.	16 Nov. 21	
159	+42·4390	20·5	42 20	8·9	8·5	OR	III. !!	16 Sept. 5	
160	+42·4395	22·0	42 21	8·4	8·0	OR	III.	16 Sept. 5	
161	+41·4519	25·8	41 29	9·1	8·6	OR	III. ?	16 Oct. 25	
162	+41·4530	27·5	41 45	9·2	9·0	OR	III. !	16 Aug. 30	
163	+44·4170	30·6	44 34	9·2	9·1	OR	III. !!	14 Sept. 30	
164	+37·4642	32·8	37 20	9·3	9·3	OR	III. !	22 Dec. 8	
165	+41·4577	35·8	41 43	8·5	8·0	OR	III. ?	16 Oct. 25	
166	+54·2851	41·0	54 25	8·9	..	R	III. ?	18 Oct. 12	
167	+54·2859	42·1	54 28	9·1	..	R	..	18 Oct. 12	
168	+59·2569	43·2	60 13	8·0	7·5	OR	III. !	18 Nov. 8	
169	+41·4622	45·7	41 23	8·2	7·7	OR	III.	16 Oct. 18	

No.	B.D.	R.A. 1900.	Decl.	Mags. B.D.	Mags. Obs.	Col.	Spect.	Date 1900+.	
	°	h m	° ′						
170	..	22 46·0	+44 2	..	9·2	OR	III. ?	15 Sept. 30	
171	+41·4625	46·3	42 9	9·1	9·0	OR′	III.	16 Aug. 30	
172	+41·4631	49·5	41 29	8·4	8·3	OR	III. ?	16 Oct. 25	
173	+41·4668	58·5	41 41	8·7	8·1	OR	III.	16 Sept. 1	
174	+59·2653	23 5·1	59 37	9·1	9·0	R	IV. ?	18 Nov. 11	
175	+41·4699	5·5	42 13	9·3	9·5	R	III. !!	16 Oct. 18	
176	+60·2503	10·1	61 6	9·3	9·2	R	III. !!	20 Sept. 19	
177	+59·2693	13·3	60 6	8·6	7·9	OR	III.	18 Nov. 8	
178	..	18·3	44 51	..	8·8	RR	IV. !!	14 Sept. 17	Var.
179	+44·4411	20·6	45 2	9·3	8·9	OR	III. !	14 Sept. 17	
180	..	26·5	61 34	..	9·3	R	III. !!!	20 Nov. 4	
181	+59·2743	28·3	60 3	9·0	8·7	OR	III. !!	18 Nov. 8	
182	..	33·2	41 40	..	9·5	R	IV. ?	16 Oct. 25	
183	+41·4836	35·2	41 34	9·5	9·4	R ?	..	16 Oct. 18	
184	+38·5064	39·8	38 20	9·5	9·2	R ?	..	22 Nov. 1	
185	+43·4555	44·0	43 32	9·4	9·4	OR	III. !	15 Sept. 30	
186	+60·2627	45·3	60 34	9·5	9·2	R	III.	21 Oct. 23	

Notes.

Nos. 1, 26, 122, 141, 178. See *M.N.*, **79,** 150.

28. N. of three stars.

30. Mr. Perrer (private letter). Very faint on Franklin-Adams' plate but visually 11 mag. With the 24-inch it was decided Red. Spectrum very faint but evidently banded.

43. Four obs. mag. 8·4.

81. Apex of triangle with B.D.+56°·2204, +56°·2206, and much above both.

94. F., N.G.C. 6819.

115. $2^{s}\cdot2$ P., $3\frac{1}{2}'$ N., B.D.+47°·3045.

119. f, 2′ S Krüger 4610.

140. In field N. of 79 Cygni.

180. np Hu. 1000.

186. N. star of pair.

The Red Stars and the Draper Catalogue. By Rev. T. E. Espin.

After the conclusion of the sweeps for red stars in 1899 the results, 1885 to 1899, were formed into a MS. Catalogue. The late Professor F. Krüger requested the loan of it, and it was incorporated in his "Neuer Katalog Farbiger Sterne," published as *Specola Astronomica Vaticana*, vol. vii. With the exception of a few earlier stars found with a 9-inch reflector, the observations were made with the $17\frac{1}{4}$-inch reflector, and an eyepiece with a very large field, about 45 minutes in diameter. Sir William Huggins having suggested that for mere viewing star spectra, a prism placed before the eyepiece was all that was necessary,

a spectroscope was made on this principle, the only modification being that the prism was mounted in a separate tube, and so could be made to slide to and fro, it having been found that the dispersion was increased with the distance from the eyepiece. The prism in the earlier part of the work was a direct vision one of five prisms cemented together. Later, one of three prisms of Jena glass, made by Hilger, was also used. A low-power positive eyepiece completed the spectroscope, which, being mounted in a separate adapter, at once replaced the sweeping power. The types of spectra follow the classification of Secchi, which, in the case of visual observations, as the late Professor E. C. Pickering stated, is the most convenient. The photographic method has led to great modifications in the classification, and it seemed well, now that the Draper Catalogue is completed, to compare the MS. Catalogue with it. The results are given for each hour in Table I. Column 1 gives the hour; column 2, types N, R, and S.; column 3, M(abcd); column 4, K; column 5, F; column 6, G; column 7, others of various types, viz. those marked Pec. in Draper Catalogue, some bright line stars, some called red by other observers, and not found so, and some evidently arising from misidentification, etc.; column 8 gives the number of stars not found in the Draper Catalogue; column 9, the total stars in the MS. Catalogue.

TABLE I.

Hour.	N.R.S.	M(abcd).	K.	F.	G.	Others.	Not found.	Total.
0	0	26	65	1	4	1	75	172
1	0	19	70	2	5	3	102	201
2	1	19	78	1	5	0	101	205
3	4	12	73	1	6	6	83	185
4	4	20	80	4	14	0	92	214
5	4	23	77	4	6	3	66	183
6	5	27	77	1	10	9	45	174
7	5	23	61	2	6	1	21	119
8	0	11	20	1	3	1	8	44
9	0	12	37	1	2	1	4	57
10	0	8	38	0	7	0	8	61
11	0	10	20	1	3	0	10	44
12	0	9	18	0	1	0	3	31
13	0	15	22	1	3	1	8	50
14	0	14	52	2	2	0	8	78
15	0	10	46	1	3	1	11	72
16	0	17	54	0	4	2	11	88
17	2	19	77	1	2	2	43	146
18	2	26	82	1	4	5	125	245
19	5	51	112	0	3	10	218	399
20	6	49	94	2	3	8	219	381
21	3	38	73	0	1	2	158	275
22	1	28	95	0	3	2	153	282
23	1	39	74	1	1	1	109	226
Total	43	525	1495	28	101	59	1681	3932

From this it appears that in the MS. Catalogue out of every 100 stars 68 are of K type, 24 of M type, 6 of G or F, and 2 of N. The stars of the K type are thus the most numerous, and may be again divided up into K, K2, K5. The results are given in Table II. In the same way the type Ma, Mb, Mc, Md are given in Table III.

TABLE II.

K Stars.

Hours.	K.	K2.	K5.	Total
0–5	166	117	160	443
6–11	79	77	113	269
12–17	110	67	76	253
18–23	136	167	227	530
Total	491	428	576	1495

These numbers hardly warrant any inference, but it may be noted that the stars of K2 are fewest, and that it is only in the last group that there is a regular increase with the type.

TABLE III.

M Stars.

Hours.	Ma.	Mb.	Mc.	Md.	Total.
0–5	70	37	3	9	119
6–11	51	30	6	4	91
12–17	45	27	5	7	84
18–23	147	57	15	12	231
Total	313	151	29	32	525

In making the above analysis the red stars found subsequently to 1899 have been added.

On Perturbation of Directivity. By T. C. Hudson.

In a previous paper concerning ephemerides of small planets (*M.N.*, 84, 585) a method of analysis was indicated in which the true position in space at time t was referred to a fictitious position in a fixed referential orbit at time t_0, the logarithm of the ratios of corresponding elements of time in the two systems being denoted by ν.

In the *initial* problem, of "determining orbit" from a small set of directivity-observations made within a small interval of time, the first attempt at obtaining a value for the period is liable to large error. But, in the case of a well-observed planet, long before it becomes absolutely necessary to proceed to the refinement of taking into account the effects

of perturbations, the larger information obtained from a lengthened stretch of observational experience will or should have included such a close approximation to the value of the period that the number ν may for many years be considered as a small, slowly varying quasi-constant.

But it is not necessary to treat the alteration of ν as continuous; it is sufficient to use a constant value of ν throughout a considerable interval, of arbitrary duration, at the end of which a new value may be adopted *abruptly*, the shock of the abrupt-change being absorbed, as it were, by a corresponding abrupt-change in one and/or other of the adaptive constants $\lambda_1, \lambda_2, \lambda_3$, etc., in the expressions for λ (the perturbation of the logarithm of the radius) and for ϕ, etc. In fact, λ_2 actually arose as the integration-constant $\epsilon(-2\mu_0) - 1$. Similar considerations apply to the undirected eccentricity $E = |\mathrm{E}|$, with the remark that in general the trouble with E (unlike that with ν) is not progressive but merely periodic, and that consequently, if δE (the error of adoption in E) be so small during (say) one revolution that it may be taken up (like ν) in the adaptive constants, then change might reasonably be postponed; this old value of E, even if known to be somewhat incorrect, would still be usable for many years without serious loss of accuracy. And similarly for perihelion, with greater suavity. [The best opportunity for changing λ_1 is when $S = 0$.]

In some such way it is hoped that two procedures in two problems hitherto considered as distinct may be more and more nearly regarded as one procedure under improving conditions. I refer to determination of orbital elements from meagre information—re-determination from all past data. But this paper does not explicitly treat of the problem of determination of elements.

Now in regard to ϕ, which is one of the two addenda to θ, the referential longitude. It occurs in the relation (p. 591, ·7; contingent on $hd\mathrm{I}_\nu = 0$, p. 590; ·4)

$$|dr| = |dr_0| = d\theta + d\phi,$$

in which $|dr|$ is the |undirected| infinitesimal element of angular velocity of the directivity of the radius *on its cone*, and $|dr_0|$ is the same thing supposed to be transferred first on to a tangent plane and then on to the fixed k plane of reference.

It is clear that r will diverge sensibly from r_0; so the complete statement of longitude requires another modification, say ψ. Also, r is on the cone and not in general on the k plane of reference, so we shall require to know kr, which is the *sine of the latitude*. The formal statement of R will then be complete.

If then it be desired to obtain the longitude and sine latitude in the forms

$$\theta + \phi + \psi \quad \text{and} \quad \sin\beta = \sigma = kr,$$

we may resort to cartesian splitting and put

$$r = \epsilon \mathrm{V} k\psi \,.\, r_0 \cos\beta + k \sin\beta.$$

Then from the conditions

$$|r'| = |r_0'| = \left|\frac{dr_0}{|dr_0|}\right| = 1$$

and

$$r\,.\,\mathrm{HF}H^{-2}R^{-2} = \nabla h^1 h \text{ (p. 590; ·4)},$$

and putting

$$\beta' = \sin\chi,$$

we readily obtain

$$\cos\beta(1+\psi') = \cos\chi$$

$$\chi' + \tan\beta\cos\chi = h\mathrm{F}H^{-2} = \zeta_1 \qquad \text{say}$$

whence

$$\sigma + \sigma'' = \zeta_1(1 - \sigma^2 - \sigma'^2)^{\frac{1}{2}} = \zeta_1\cos\beta\cos\chi = \zeta_2 \qquad \text{say},$$

which is amenable to methods of successive approximation. The prime variable here is $\theta + \phi$, not θ.

Thus we see that as the natural argument for ϕ is θ, so the natural argument for sin β, β', χ, ψ' and ψ is $\theta + \phi$, and I do think this natural dependence should be respected in procedure. The fact that the appropriate argument for final assertions (ephemerides) is date does not at all imply that formal and somewhat auxiliary statements should be temporal. In the case of the moon (which is, of course, a particular case of the F equation, p. 589, ·6), the number of terms depending on the time is becoming so formidable as to begin to impair the accuracy of ephemeris by sheer accumulation of the unavoidable errors of "curtation," as Newcomb called them; and it is becoming increasingly difficult to locate the further errors of actual computation, to say nothing of the huge labour of synthesis from beginning to end. Now, the resources of the calculus of finite differences are not nearly exhausted. Given appropriate arguments for tabulation, much use might be made of functions of finite differences (*e.g.* products) to simplify the task of the executive in ephemeris offices, who translate symbol into digit.

There are a few questions which have not been touched:—

(1) Propriety of choice of origin.
(2) The exponential setting of Hamilton's linear vector operator.
(3) The utility of the equation of directed eccentricity.

(1) *The Origin.*—The equation

$$(1 + R^3(d/dt)^2)\mathrm{R} = \mathrm{F} \text{ (p. 589, ·6)}$$

contains little or no evidence that the choice of origin is commendable. The point of the discussion may be obscured by appeal to the case F = 0, because in that case the origin is in line with both R and ^{2}R.

But if we take the *general* equation of directed acceleration: $^2\mathrm{R}$ = some occult single-valued function of R of form unknown, then where are the credentials of the origin of R? All we are really given is that at any instant the point whose path is traced by the other end of R has a definable directed acceleration ^{2}R; assuming that the origin is either "at rest or moving uniformly in a straight line," we see that ^{2}R contains no information whatever as to the location of the origin, and

all we have to go on is that we are given information as to a definite directed magnitude associated with each and any position of the moving point.

And if we persist in demanding an origin that shall be relevant to the case, we are bound to consider this line $^2\mathrm{R}$ that begins at the moving point and ends somewhere else, because there is nothing else to consider; if the good origin does not lie on the located line $^2\mathrm{R}$, then where ought it to lie? Hence it ought to lie on $^2\mathrm{R}$. Then whereabouts on $^2\mathrm{R}$? At the beginning? At the point itself? No, that would only give the egoistic solution $0=0$. Thus one choice is ruled out: the moving point itself.

Then which point on $^2\mathrm{R}$? If we choose the other end of $^2\mathrm{R}$ we know that the choice is in *general* inappropriate, because it is inappropriate in *particular* in the case of $\mathrm{R}+{}^2\mathrm{R}R^3=0$.

Thus there must be at least one point on $^2\mathrm{R}$ which has special credentials, and in general it is at neither end; but no more can be said at present.

Let us examine the $^2(\ )$ of $^2\mathrm{R}$. It denotes a *repetition* of the operator d/dt. Thus $^1\mathrm{R}$ the directed velocity is indicated as the only clue remaining.

[At this stage it is unnecessary, but it may be convenient to assume that the arbitrary and therefore irrelevant origin has no velocity, or, alternatively, that a fixed origin has been substituted and the necessary transformations effected.]

$^1\mathrm{R}$ may be taken to mean either that after an infinitesimal time dt the moving point will have left the end of R and arrived at the end of $\mathrm{R}+d\mathrm{R}$, or else that at time dt previously it was at $\mathrm{R}-d\mathrm{R}$. $d\mathrm{R}$ is as unknown as R, of course, but if we *did* know R and $\mathrm{R}+d\mathrm{R}$ we should know $^2\mathrm{R}$ and $^2(\mathrm{R}+d\mathrm{R})$. Thus, in virtue of the given equation, in a *conditional* sense and meaning we may fairly be said to know two consecutive lines.

Now, each of these lines has one point or pair of coincident points, and one only, that is nearest to the other line. And if we get rid of the ambiguity, as to past or future, by taking three values of $^2\mathrm{R}$ at equidistant intervals of time dt, we obtain this precept for procedure:

Take the nearest, on the middle line, to the join of the nearests on the first and third.

In this way we arrive at a *reasoned* choice of origin. Starting from the moving point, this reasoned origin is at the end of

$$^2\mathrm{R}\frac{v^1\mathrm{R}^2\mathrm{R}v^2\mathrm{R}^3\mathrm{R}}{(v^2\mathrm{R}^3\mathrm{R})^2}.$$

(For v see p. 586, Tab. IV.)

This origin has unique credentials, then, in the general case. And they are obviously satisfactory in the particular case

$$(1+R^3(d/dt)^2)\mathrm{R}=0,$$

and still more obviously in the case

$$\left(1+\left(\frac{d}{d\theta}\right)^2\right)r=0.$$

Why should these credentials suddenly fail to be valid in the sub-general case of the F equation, the case that occurs in dynamical astronomy?

We can even make an illusory perturbation by deliberate mis-choice of origin—say the wrong focus; we then get an illusory F equation. It is true that if we accepted the origin without protest and proceeded to use known methods of calculating perturbations, we should ultimately obtain a solution not much at variance with the facts. But if we adopted this general method of investigation of the origin by means of ${}^1\mathrm{R}^2\mathrm{R}^3\mathrm{R}$, we should soon find and identify the proper focus awaiting us.

In fact, we could devise as many spurious F equations as we liked. If we had a sufficient number of them, we might possibly arrive at data for a solution of the general F equation. Be that as it may, we could at any time utilise illusory cases of the F equation for the purpose of testing the correctness of generalised algebraic results in F.

As a matter of fact, it was by the use of this point of serenity on ${}^2\mathrm{R}$ that the λ, θ solution (p. 592) was actually arrived at before the λ, θ equation itself was written.

And if this point was useful for parameters of scale, to which it is not obviously relevant, may it not be useful for parameters of directivity, to which it is more closely related?

The writer does not propose to answer this question, but to refer to page 589, line 6 up: "K^2 depends on the epoch chosen," and also to refer to a lunation when Venus terms of Sun's Latitude countervailed the Moon terms.

(2) *The Exponential Setting of Hamilton's Linear Vector Operator.*—The operator $\epsilon\nu\alpha(\gamma+\gamma)$ is appropriate to the case of *turn of a rigid system*, whereas r is not a rigid system, but merely a single directivity. But any two of the three directivities r, $dr/|dr|$, and $vrdr/|dr|$ make a rigid system, and together they correspond to "the radial, the transversal, and the orthogonal" components of the perturbing force. Formulæ for (p. 590) conjoint turn:

$$\mathrm{S}=\mathrm{S}_1C_2+\mathrm{S}_2C_1+v\mathrm{S}_1\mathrm{S}_2,\ C=C_1C_2-\mathrm{S}_1\mathrm{S}_2,\ \mathrm{T}(1-\mathrm{T}_1\mathrm{T}_2)=\mathrm{T}_1+\mathrm{T}_2+v\mathrm{T}_1\mathrm{T}_2.$$

The operator which relates r to (k, p_0) can be expressed with triple products of natural sines and cosines of three semiangles.

(3) *The Utility of the Directed Equation of Eccentricity.*—When I showed the eccentricity equation to Professor Plummer, he immediately suggested application to perturbations. But I have not sounded the possibilities. Of course, the $1+E\cos\theta$ in the λ equation (p. 592) is really $r(r+\mathrm{E})$.

The Transit of Mercury across the Sun's Disc, 1924 *May* 7, *as observed at the University Observatory, Simferopol, Crimea.* By Professor N. Liapin.

(*Communicated by the Secretaries.*)

The transit of Mercury was observed here under very favourable conditions. Four observers took part in the observations—E. Perepelkin (P.), D. Martynoff (M.), M. Eigenson (E.), and N. Tchijevsky (T.). The approximate co-ordinates of the Observatory are—

Longitude from Greenwich . . .	$2^h\ 16^m\ 26^s$ E.
Latitude	$44^\circ\ 57'{\cdot}1$ N.
Altitude above the sea-level . . .	280 m.

A. 110 mm. Zeiss refractor. Observer P.

(α) The diameter of Mercury was measured by means of the breadth of spectrum as follows :—

1924 May 7, 17^h G.M.T. $d = 11''{\cdot}68 \pm 0''{\cdot}23$.

In the spectrum of Mercury, except lines of solar spectrum in the red-yellow and green, no lines could be distinguished.

The spectrometer used was a differential one of Reinfelder and Hertel. In the green part of the spectrum the dispersion was about 150 Å. to the millimetre. The equivalent power of the system refractor-spectrometer was 93. A reddish glass-filter was used.

(β) The black drop phenomenon was observed as follows :—

1924 May 7, $17^h\ 35^m\ 56^s$ G.M.T. beginning
,, 17 36 10 ,, end.

(γ) Times of the third and fourth contacts—

1924 May 7, $17^h\ 36^m\ 0^s$ G.M.T.
,, 17 38 39 ,,

respectively. Seeing fairly good (6).

110 mm. Zeiss refractor. Observer M.

(α) The black drop phenomenon was clearly observed.

(β) Times of the third and fourth contacts—

1924 May 7, $17^h\ 36^m\ 17^s$ G.M.T.
,, 17 38 54 ,,

By both observers the projection on a screen was used. Diameter of the Sun was 50 cm., of Mercury 3 mm. Power 124. Huygens eye-piece. Time was recorded with Ericsson pocket-chronometer (P.) and marine Ericsson chronometer (M.).

B. 81 mm. Reinfelder and Hertel refractor. Observer E.

(α) No black drop phenomenon was seen.

(β) Mercury was considerably darker than sunspots.

(γ) Times of the third and fourth contacts—

1924 May 7, $17^h\ 35^m\ 29^s$ G.M.T.
„ 17 38 30 „

81 mm. Reinfelder and Hertel refractor. Observer T.

(α) No black drop phenomenon was observed.

(β) Times of the third and fourth contacts—

1924 May 7, $17^h\ 35^m\ 34^s$ G.M.T.
„ 17 38 37 „

By both observers the projection on a screen was used. Diameter of the Sun was 15 cm., of Mercury 1 mm. Power 55. The time was recorded with ordinary pocket-watch. Seeing good.

The times of the third and fourth contacts as calculated for Simferopol from the formulæ given in the American Ephemeris for 1924 were—

1924 May 7, $17^h\ 36^m\ 19^s{\cdot}4$ G.M.T.
„ 17 39 18·6 „

respectively.

University Observatory, Simferopol, Crimea:
1924 *June* 15.

Occultation of Venus by the Moon, observed on 1924 *September* 24 *at the Astronomical Observatory, Kiew.* By Professor S. D. Tscherny.

(*Communicated by the Secretaries.*)

	Greenwich Mean Time. h m s	Telescope and Power.	Observer.	Remarks.
Contact of S cusp of Venus and of the bright limb of the Moon.	13 36 31·33	G_{58}	K	Good.
Contact of S cusp of Venus and of the bright limb of the Moon.	13 36 31·70	$R-H_{89}$	P	
Contact of the centre of Venus and of the bright limb of the Moon.	13 36 35·89	U_{74}	M	
Contact of the bright limb of Venus and of the bright limb of the Moon.	13 37 3·16	R_{30}	T	Good.
Contact of the bright limb of Venus and of the bright limb of the Moon.	13 37 3·74	G_{58}	K	Good.

	Greenwich Mean Time. h m s	Telescope and Power.	Observer.	Remarks.
Reappearance of S cusp of Venus on the dark limb of the Moon.	14 47 55·51	R_{90}	T	
Reappearance of S cusp of Venus on the dark limb of the Moon.	14 47 55·54	G_{58}	K	Good.
Reappearance of S cusp of Venus on the dark limb of the Moon.	14 47 55·18	$R-H_{89}$	P	
Contact of the bright limb of Venus and of the dark limb of the Moon.	14 48 22·44	R_{90}	T	Good.
Contact of the bright limb of Venus and of the dark limb of the Moon.	14 48 21·47	G_{58}	K	A little earlier.

Instruments used :—

R = 243 mm. Merz-Repsold refractor.
R – H = 107 mm. Reinfelder und Hertel refractor.
G = 70 mm. Goerz refractor.
U = 72 mm. Utzschneider und Fraunhofer refractor.

Observers :—

T = S. Tscherny.
K = A. Karamischev.
P = N. Pisarenko.
M = A. Muschkin.

Astronomical Observatory, Kiew :
1924 *October* 6.

Occultations of Stars by the Moon observed at the Observatory of the University of Tomsk ($\phi = 56° 28' 07''$; $\lambda = 5^h 39^m 47^s·4$ *E. from Greenwich*), *during the years* 1923–24. By Professor N. Goryatchew.

(*Communicated by the Secretaries.*)

Date.	*		Phase.	Local Sidereal Time. h m s
1923.				
IV. 19	γ Tauri	3·9	D	10 12 16·3
V. 26	θ Virginis	4·4	D	17 39 16·0
VIII. 6	71_1 Tauri	4·6	R	23 39 30·1
	θ^2 Tauri	3·6	D	23 54 18·0
	θ^1 Tauri	4·2	D	00 01 41·0
19	49 Libræ	5·4	D	18 02 45·7
31	389 B Ceti	6·3	R	22 38 05·5
IX. 4	130 Tauri	5·6	R	00 02 09·6
18	B.D. – 19° 5134	6·5	D	20 45 33·5
20	47 B Capricorni	6·2	D	21 44 55·2

Date.	*		Phase.	Local Sidereal Time. h m s
1923.				
X. 1	111 Tauri	5·1	R	22 51 00·8
	117 Tauri	6·0	R	00 35 09·2
29	124 H^1 Orionis	5·7	R	08 17 30·2
1924.				
II. 8	4 Ceti	6·3	D	02 29 57·2
	5 Ceti	6·3	D	02 50 09·5
	19 B Ceti	7·3	D	03 27 03·3
9	167 B Piscium	7·3	D	04 04 34·5
13	179 B Tauri	5·9	D	05 33 25·6
14	318 B Tauri	5·7	D	06 13 30·2
23	46 Virginis	6·1	R	10 16 43·4
	48 Virginis	6·5	R	12 06 35·3
24	598 B Virginis	6·1	R	10 09 04·5
	614 B Virginis	6·6	R	12 50 03·4
	94 Virginis	6·6	R	16 34 09·3
26	204 B Libræ	6·9	R	15 19 28·1
IV. 14	35 B Leonis	6·7	D	12 39 36 3
VI. 8	21 Leonis	6·7	D	14 52 13·9
22	χ Aquarii	5·3	R	19 50 20·4
VII. 22	117 G Piscium	6·5	R	22 55 49·7
26	89 Tauri	5·8	D	23 04 36·2
	89 Tauri	5·8	R	23 35 11·2
27	115 Tauri	5·3	D	22 59 54·3
	115 Tauri	5·3	R	23 49 47·1

These occultations were observed by Professor Goryatchew; instrument: telescope (Zeiss) with diameter of objective = 90 mm. and power of ocular = 108.

The probable error of local sidereal time = $\pm 0^s \cdot 2$.

The University Observatory,
S.S.S.R. Siberia, Tomsk:
1924 *September* 27.

Erratum.

In connection with the paper by Messrs. Ross and Thompson on the eclipse of the sun on 1926 January 14 (*M.N.*, 84, 662), Mr. Z. A. Merfield, of Melbourne University, has pointed out that the local mean time of mid-eclipse given by Professor Ross for the first station should be $2^h\ 24^m \cdot 1$, and not $2^h\ 10^m \cdot 1$.

It might also be mentioned here that in the above paper the letters G.M.T. refer to the new system of Greenwich time beginning at midnight, and that the times given as L.M.T. should be increased by twelve hours to correspond to this.